Growth of COVID-19 cumulative data in italian regions.

Max Pierini

info@maxpierini.it
17 agosto 2020

1 ABSTRACT

A simple model is presented to define the growth type of COVID-19 cumulative data (total cases and deaths) during the last 14 days in italian regions.

Data are supposed to follow 4 different growths:

- None: constant data with no growth
- Linear: linear growth
- Exponential: exponential growth
- Exp. Decay: exponential increasing decay

A unified exponential curve function will be used, capable of describing the four growths with a Markov chain Monte Carlo.

2 DATA

Official data from **Dipartimento di Protezione Civile** GitHub repository will be used for cases `totale_casi` and deaths `deceduti` cumulative data time series.

Data will be used raw "as is", with no smoothing and/or corrections.

3 METHOD

The last $w = 14$ days of observed cumulative y_r data (cases and deaths) of each r region will be fitted to a unified exponential curve

$$\mathbb{E}[y_r] = \frac{e^{\beta_r x} - 1}{e^{\beta_r} - 1}(y_{r,w} - y_{r,1}) + y_{r,1}$$

where $y_{r,1}$ is the first and $y_{r,w}$ the last y_r observed values in time window w; $\beta_r \in \mathbb{R}$ and $x = [0...1]$ with $|x| = w$.

Since

$$\frac{e^{\beta x} - 1}{e^{\beta} - 1} = \frac{1 - e^{\beta x}}{1 - e^{\beta}}$$

and

$$\lim_{\beta \to 0} \frac{e^{\beta x} - 1}{e^{\beta} - 1} = x$$

- for $\beta_r > 0$ the curve is an exponential growth
- for $\beta_r < 0$ the curve is an exponential decay
- for $\beta_r \xrightarrow{\lim} 0$ the curve is linearly growing

The curve has no solutions if y_r is constant thus if β_r posteriors are not significantly different from priors, observed y_r will be supposed constant.

The proposed unified exponential equation $\mathbb{E}[y_r]$ is believed to be capable of simultaneously estimate the best fit for linear, exponential growth and exponential decay on every time window w of cumulative raw data, with no need to smooth them, regardless of initial $y_{r,1}$ and final $y_{r,w}$ data. Thus, it can highlight an onset of potentially dangerous exponential growth or favourable exponential decay in any cumulative data time window, with a 99% bayesian credible interval.

Parameters β_r could be virtually unbounded below and above, but for $x = [0...1]$ and $|x| = w$ a limit of β can be defined

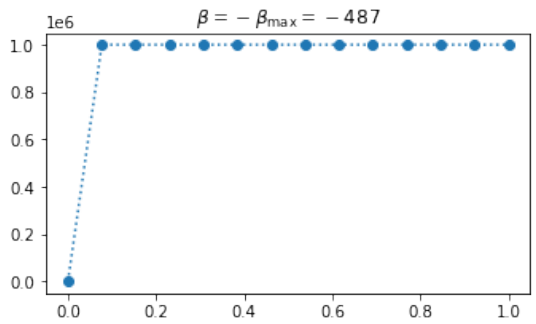
$$\beta_r \geq \beta_{\max} \Rightarrow \begin{cases} \mathbb{E}[y_r]_{w-2} \simeq \mathbb{E}[y_r]_{1...w-3} & \text{if } \beta_r > 0 \\ \mathbb{E}[y_r]_2 \simeq \mathbb{E}[y_r]_{3...w} & \text{if } \beta_r < 0 \end{cases}$$

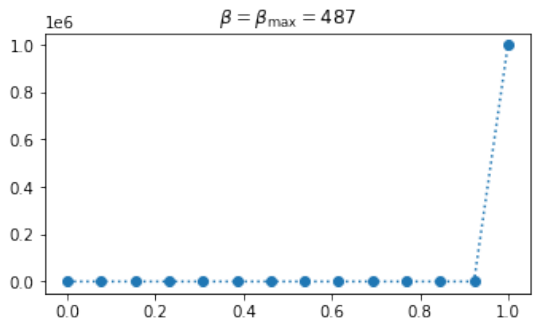
For $y_1 = 1$ and taking a maximum of $y_w = 1,000,000$ in $w = 14$ days, $\beta_{\max} = 487$ thus, to ensure data driven posteriors, parameters β_r will be distributed **Uniform** between -487 and +487

$$\beta_r \sim \mathcal{U}(-487, 487)$$

In python (where `exp_unif` is $\mathbb{E}[y]$ function):

```
yLo = 1
yHi = 1e6
beta_lim = None

for b in np.arange(1000):
    if not b:
        continue
    yExp, _ = exp_unif(x, b, yLo, yHi)
    yDec, _ = exp_unif(x, -b, yLo, yHi)
    if (yDec[1:]==yHi).all() and (yExp[:-2]==yLo).all():
        beta_lim = b
        break
```





Observed y_r will be distributed **Normal** with mean equal to $\mathbb{E}[y_r]$ and unknown precision τ_r

$$y_r \sim \mathcal{N}\Big(\mathbb{E}[y_r], \tau_r\Big)$$

where τ_r are distributed as uniformative **Gamma**

$$\tau_r \sim \Gamma(0.001, 0.001)$$

so that variances σ_r^2 are distributed as uniformative **Inverse Gamma**

$$\sigma_r^2 \sim \Gamma^{-1}(0.001, 0.001)$$

and standard deviations σ_r are

$$\sigma_r = \frac{1}{\sqrt{\tau_r}}$$

A Markov chain Monte Carlo with Metropolis-Hasting algorithm and Gibbs sampler will be used (`JAGS`: adapt 2000, iter 8000, thin 1, chains 4).

If observed y_r are constant, $\beta_{r,\text{post}}$ posteriors won't differ from $\beta_{r,\text{prior}}$ priors.

Difference from priors will be defined by a Bartlett's test for equal variances between β_{prior} priors samples and $\beta_{r,\text{post}}$ posteriors samples, rejecting null hypotesis that all input samples are from populations with equal variances for a $p-\text{value} < .05$.

If β_r posteriors differ enough from priors and β_r HPDI 99% contains zero, the credibility interval 99% does not exclude linear growth, so data will be supposed to follow a linear growth with α_r slope equal to

$$\alpha_r = \frac{y_{r,w} - y_{r,1}}{w - 1}$$

otherwise, for β_r posteriors greater than zero data will likely follow exponential growth or exponential decay for β_r posteriors lower than zero.

$$\text{growth}_r = \begin{cases} \beta_{r,\text{post}} \simeq \beta_{r,\text{prior}} & \text{No growth} \\ 0 \in \beta_{r,\text{post}} & \text{Linear} \\ \beta_{r,\text{post}} < 0 & \text{Exp. Decay} \\ \beta_{r,\text{post}} > 0 & \text{Exponential} \end{cases}$$

Errors in β_r parameters could occur if cumulative data are decreasing (due to data error corrections), i.e. $y_{r,1} > y_{r,w}$. Theese results will be treated as errors in summary.

3.1 Model

Model sketch:

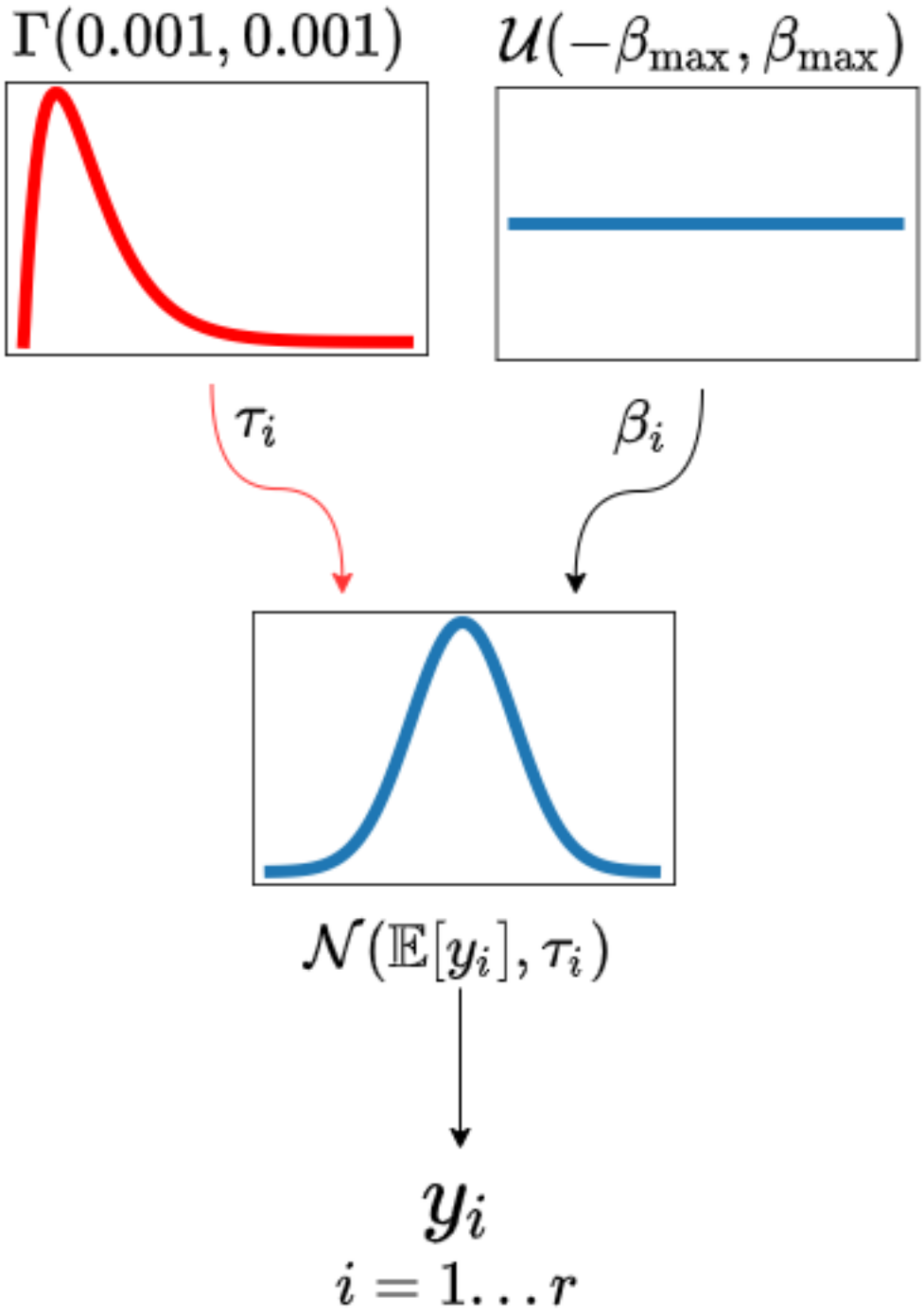


Figura 1: model

Model data:

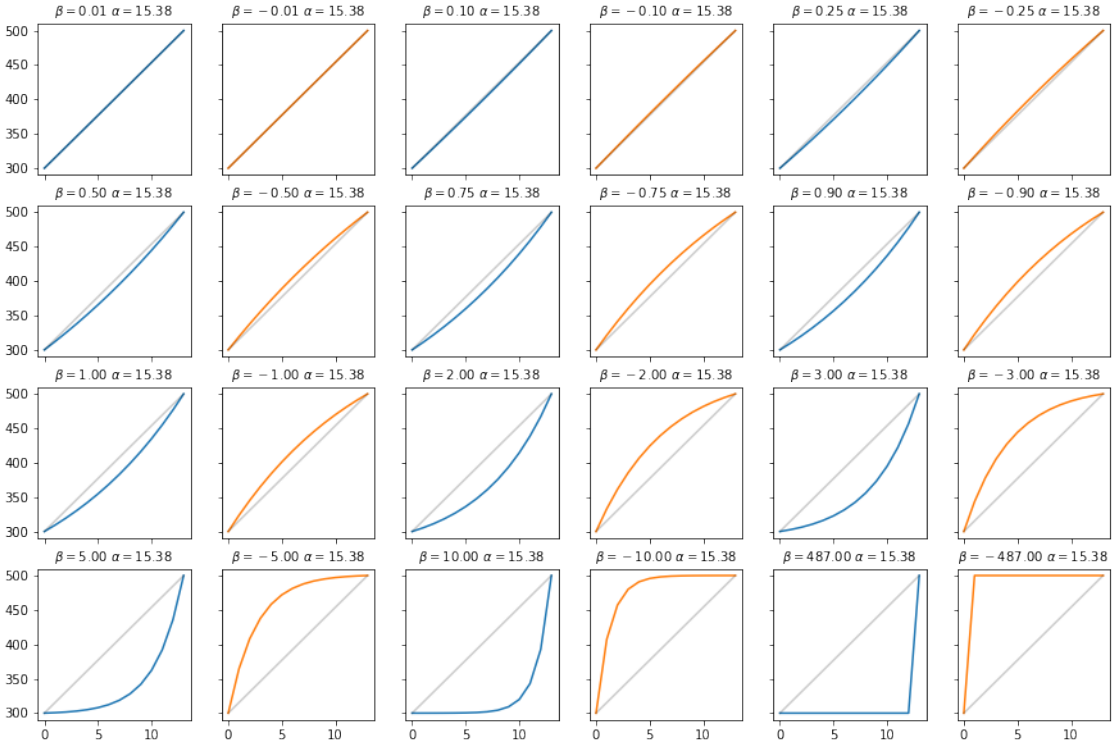
```
R  = int  # integer:     Number of regions
W  = int  # integer:     Time window to fit
x  = []   # array[W]:    Values of x = 0..1
y  = [[]] # matrix[R,W]: Observed y data
y1 = []   # array[R]:    First observed data
yw = []   # array[R]:    Last observed data
```

Model in JAGS:

```

model {
  for ( r in 1:R ) {
    # inverse gamma variance
    tau[r] ~ dgamma( 0.001 , 0.001 )
    sigma[r] <- 1 / sqrt( tau[r] )
    # beta priors
    beta[r] ~ dunif( -487 , 487 )

    for ( t in 1:W ) {
      # define lim(beta->0)
      H[r,t] <- ( exp( beta[r] * x[t] ) - 1 )
      f[r,t] <- ifelse( nu[r] == 1 , x[t] , H[r,t] / nu[r] )
      # expected and observed
      E[r,t] <- f[r,t] * (yw[r] - y1[r]) + y1[r]
      y[r,t] ~ dnorm( E[r,t] , tau[r] )
    }

    # define lim(beta->0)
    nu[r] <- ifelse( exp( beta[r] ) - 1 == 0 , 1 , exp( beta[r] ) - 1 )
  }
}
```

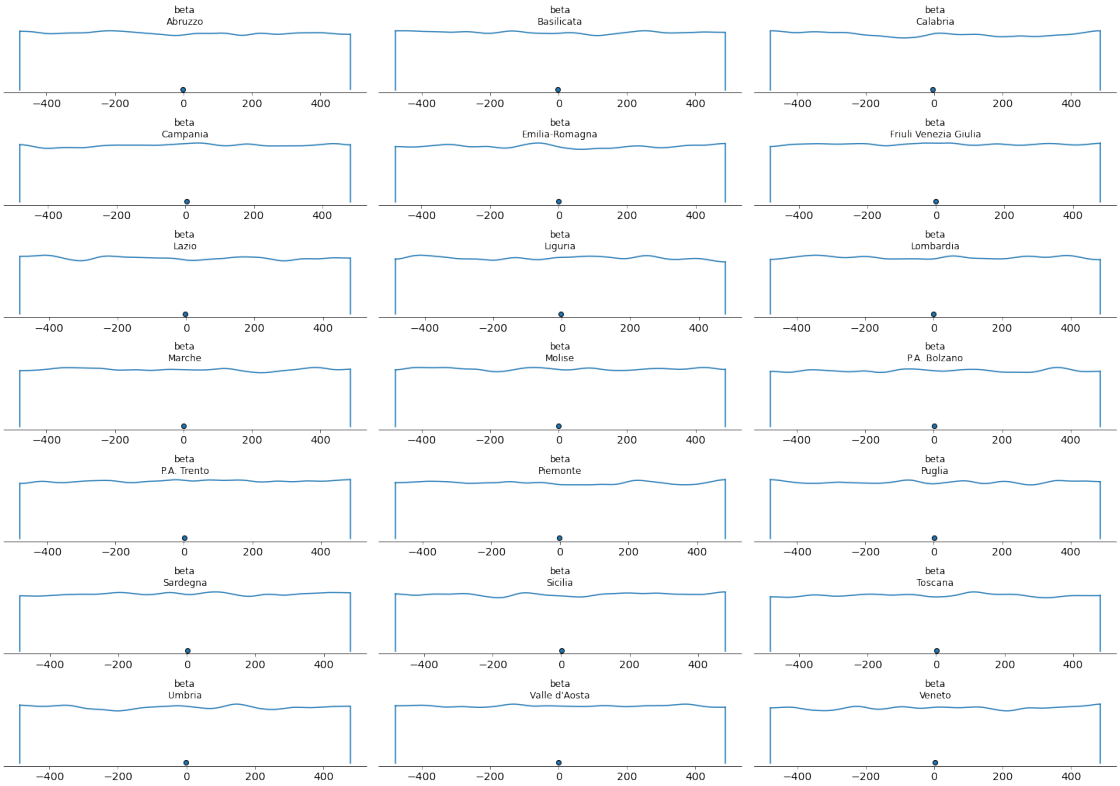
3.2 Examples of Unified Exponential Curve

Examples for $y_1 = 300$ and $y_w = 500$ with $w = 14$.


$\beta = 0.01$ $\alpha = 15.38$
$\beta = -0.01$ $\alpha = 15.38$
$\beta = 0.10$ $\alpha = 15.38$
$\beta = -0.10$ $\alpha = 15.38$
$\beta = 0.25$ $\alpha = 15.38$
$\beta = -0.25$ $\alpha = 15.38$
$\beta = 0.50$ $\alpha = 15.38$
$\beta = -0.50$ $\alpha = 15.38$
$\beta = 0.75$ $\alpha = 15.38$
$\beta = -0.75$ $\alpha = 15.38$
$\beta = 0.90$ $\alpha = 15.38$
$\beta = -0.90$ $\alpha = 15.38$
$\beta = 1.00$ $\alpha = 15.38$
$\beta = -1.00$ $\alpha = 15.38$
$\beta = 2.00$ $\alpha = 15.38$
$\beta = -2.00$ $\alpha = 15.38$
$\beta = 3.00$ $\alpha = 15.38$
$\beta = -3.00$ $\alpha = 15.38$
$\beta = 5.00$ $\alpha = 15.38$
$\beta = -5.00$ $\alpha = 15.38$
$\beta = 10.00$ $\alpha = 15.38$
$\beta = -10.00$ $\alpha = 15.38$
$\beta = 487.00$ $\alpha = 15.38$
$\beta = -487.00$ $\alpha = 15.38$
500
450
400
350
300
0
5
10

4 PRIORS

4.1 beta



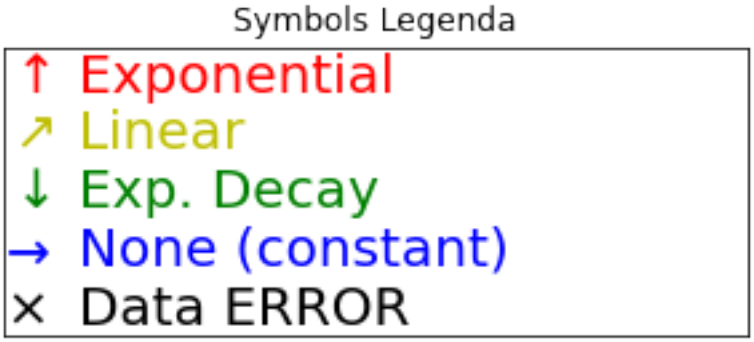
	mean	sd	hdi_0.5%	hdi_99.5%
Abruzzo	-2.129	281.433	-476.795	486.608
Basilicata	-1.794	282.453	-477.385	486.814
Calabria	-3.417	283.950	-478.245	485.259
Campania	3.331	279.442	-480.864	480.460
Emilia-Romagna	0.926	282.044	-476.631	486.277
Friuli Venezia Giulia	-0.063	280.676	-483.353	480.271
Lazio	-2.974	282.159	-484.540	479.419
Liguria	-2.255	280.481	-486.573	477.281
Lombardia	0.020	281.283	-476.984	486.508
Marche	-0.626	281.448	-476.797	486.448
Molise	0.035	281.582	-477.331	486.167
P.A. Bolzano	1.317	280.690	-477.157	486.323
P.A. Trento	1.592	280.653	-478.547	485.384
Piemonte	-1.550	281.774	-480.079	481.865
Puglia	0.419	281.979	-477.650	486.701
Sardegna	1.634	280.663	-486.789	476.925

Continued on next page

	mean	sd	hdi_0.5%	hdi_99.5%
Sicilia	1.976	282.128	-484.369	479.285
Toscana	0.335	279.478	-484.218	479.282
Umbria	-0.653	282.037	-485.443	478.014
Valle d'Aosta	0.442	281.097	-477.111	486.940
Veneto	2.416	281.907	-478.615	485.429

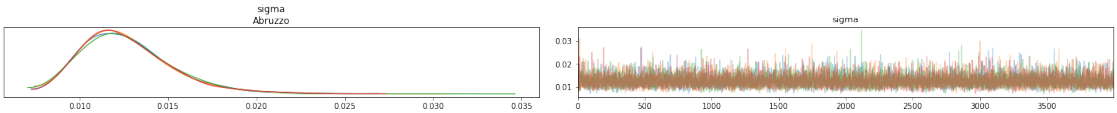
	mcse_mean	mcse_sd	ess_mean	ess_sd	ess_bulk	ess_tail	r_hat
Abruzzo	2.177	1.609	16717.0	15302.0	16456.0	15953.0	1.0
Basilicata	2.213	1.568	16284.0	16227.0	16532.0	16200.0	1.0
Calabria	2.278	1.611	15534.0	15534.0	15672.0	15126.0	1.0
Campania	2.261	1.599	15272.0	15272.0	15401.0	15723.0	1.0
Emilia-Romagna	2.227	1.577	16041.0	15996.0	16170.0	15808.0	1.0
Friuli Venezia Giulia	2.264	1.601	15374.0	15374.0	15515.0	15650.0	1.0
Lazio	2.247	1.589	15769.0	15769.0	15724.0	16100.0	1.0
Liguria	2.221	1.570	15952.0	15952.0	15835.0	15649.0	1.0
Lombardia	2.242	1.591	15737.0	15624.0	16009.0	16126.0	1.0
Marche	2.265	1.602	15435.0	15435.0	15289.0	15376.0	1.0
Molise	2.200	1.564	16376.0	16207.0	16261.0	15770.0	1.0
P.A. Bolzano	2.229	1.577	15856.0	15833.0	15766.0	15765.0	1.0
P.A. Trento	2.235	1.625	15773.0	14919.0	15903.0	15354.0	1.0
Piemonte	2.235	1.580	15899.0	15899.0	15872.0	15792.0	1.0
Puglia	2.214	1.566	16227.0	16207.0	16125.0	15720.0	1.0
Sardegna	2.235	1.580	15768.0	15768.0	15723.0	14983.0	1.0
Sicilia	2.225	1.573	16075.0	16075.0	16117.0	15440.0	1.0
Toscana	2.202	1.557	16102.0	16102.0	16124.0	16165.0	1.0
Umbria	2.226	1.624	16047.0	15077.0	15916.0	15392.0	1.0
Valle d'Aosta	2.216	1.585	16096.0	15736.0	16103.0	15250.0	1.0
Veneto	2.243	1.593	15798.0	15655.0	15832.0	14935.0	1.0

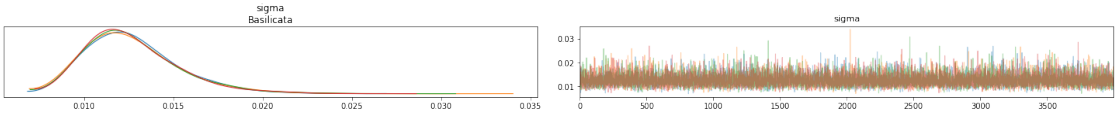
5 RESULTS

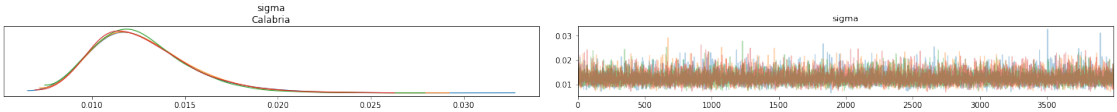


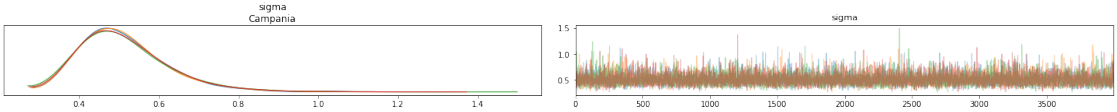
5.1 deceduti

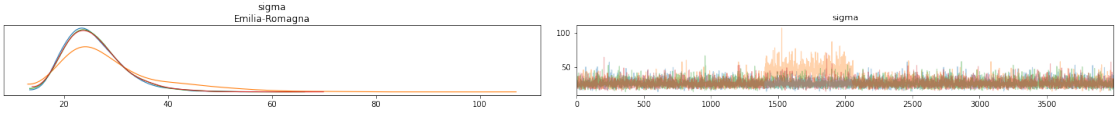
5.1.1 sigma

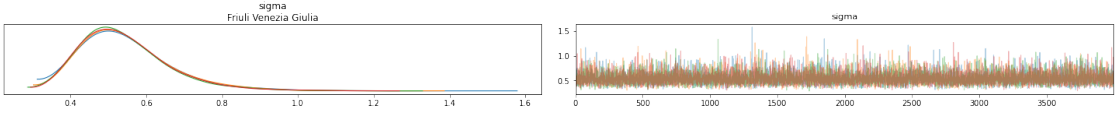


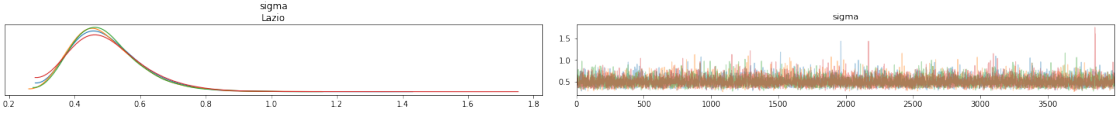


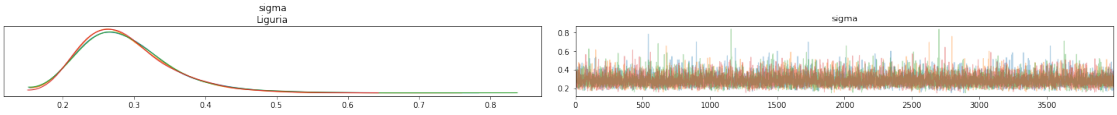


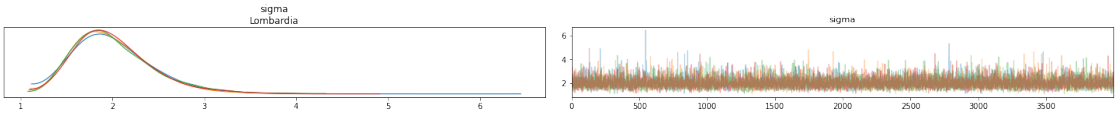


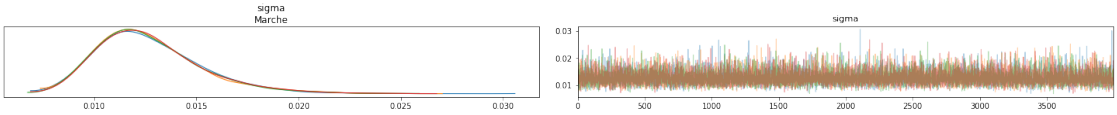


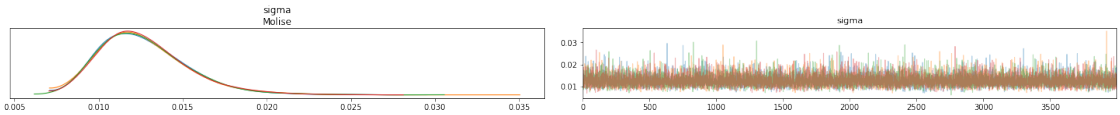


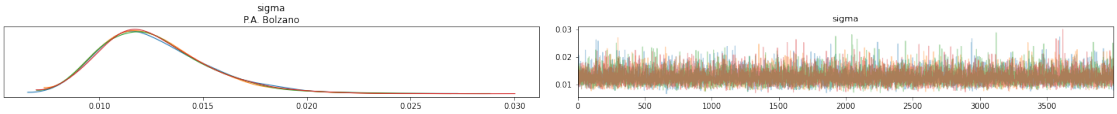


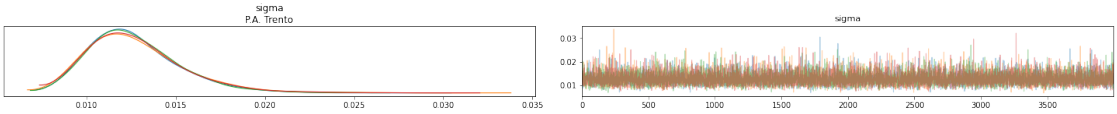

sigma
Liguria
sigma

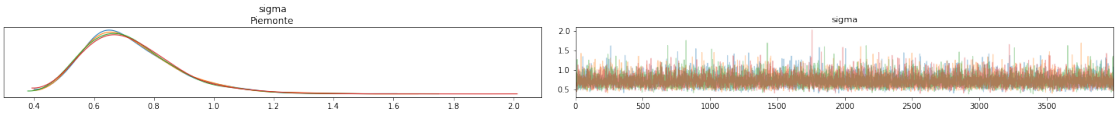

sigma
Lombardia
sigma

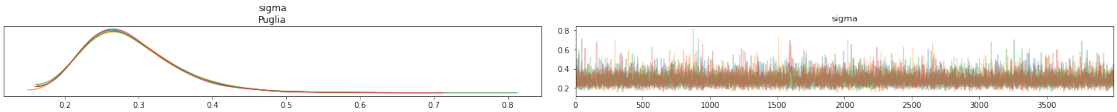

sigma
Marche
sigma

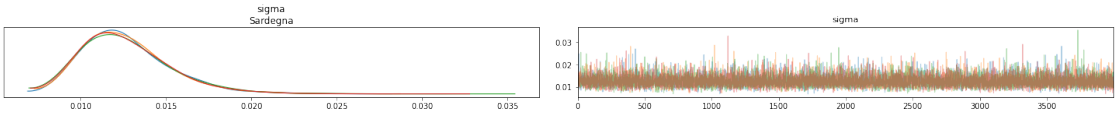

sigma
Molise
sigma

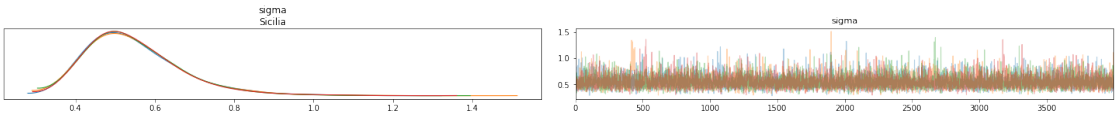

sigma
P.A. Bolzano
sigma

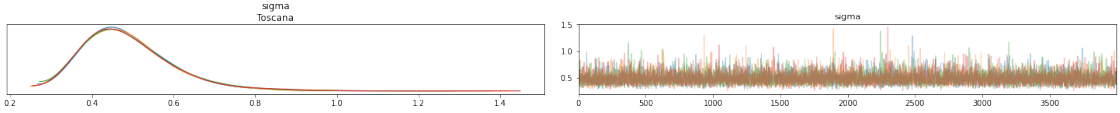

sigma
P.A. Trento
sigma

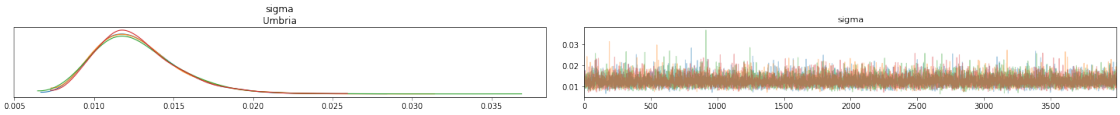

sigma
Piemonte
sigma

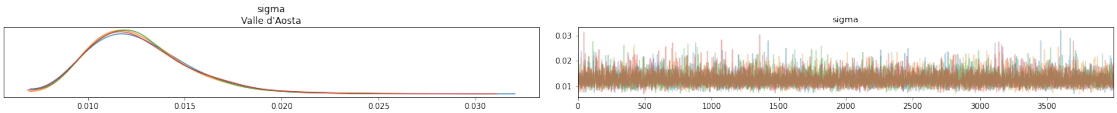

sigma
Puglia
sigma

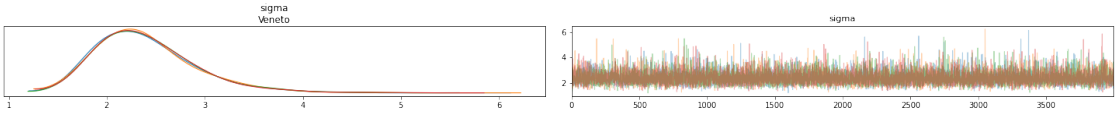

sigma
Sardegna
sigma

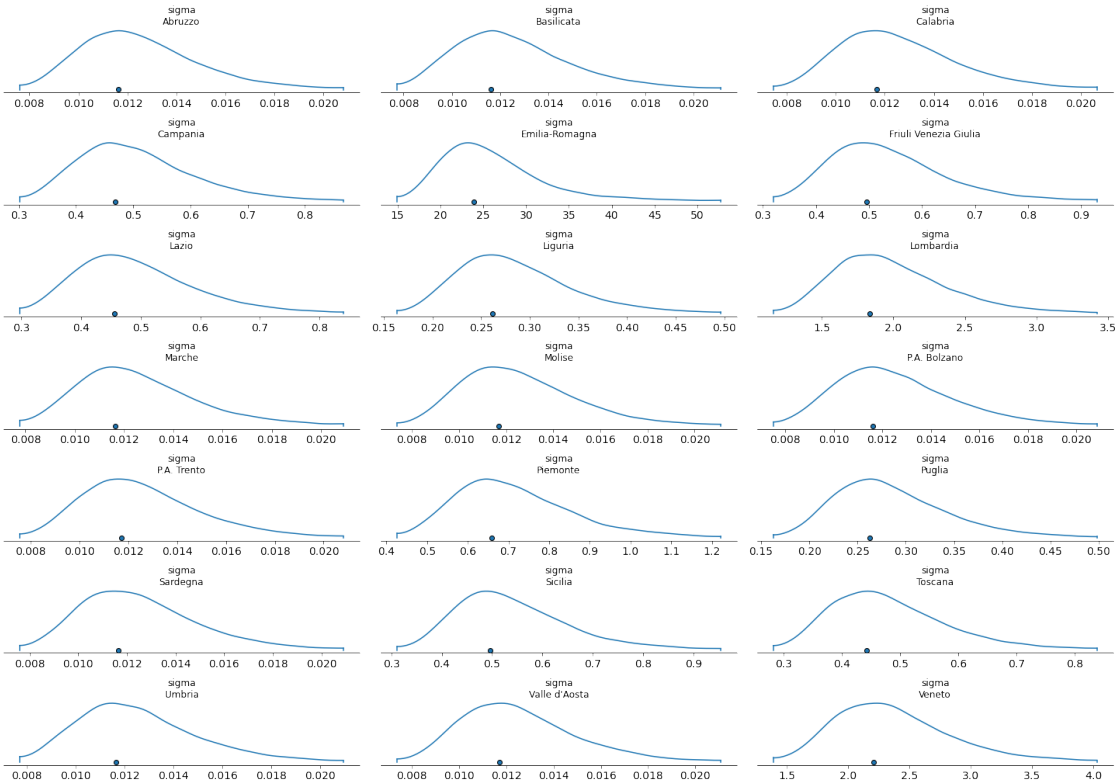

sigma
Sicilia
sigma

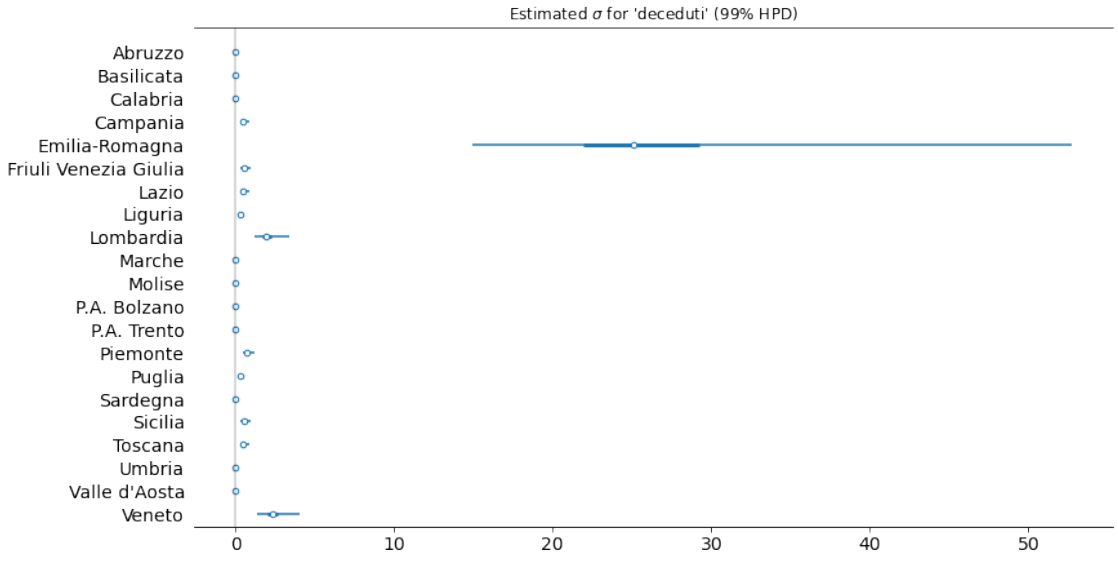

sigma
Toscana
sigma


sigma
Umbria
sigma


sigma
Valle d'Aosta
sigma


sigma
Veneto
sigma


sigma
Abruzzo
sigma
Basilicata
sigma
Calabria
sigma
Campania
sigma
Emilia-Romagna
sigma
Friuli Venezia Giulia
sigma
Lazio
sigma
Liguria
sigma
Lombardia
sigma
Marche
sigma
Molise
sigma
P.A. Bolzano
sigma
P.A. Trento
sigma
Piemonte
sigma
Puglia
sigma
Sardegna
sigma
Sicilia
sigma
Toscana
sigma
Umbria
sigma
Valle d'Aosta
sigma
Veneto


Estimated σ for 'deceduti' (99% HPD)
Abruzzo
Basilicata
Calabria
Campania
Emilia-Romagna
Friuli Venezia Giulia
Lazio
Liguria
Lombardia
Marche
Molise
P.A. Bolzano
P.A. Trento
Piemonte
Puglia
Sardegna
Sicilia
Toscana
Umbria
Valle d'Aosta
Veneto
0
10
20
30
40
50

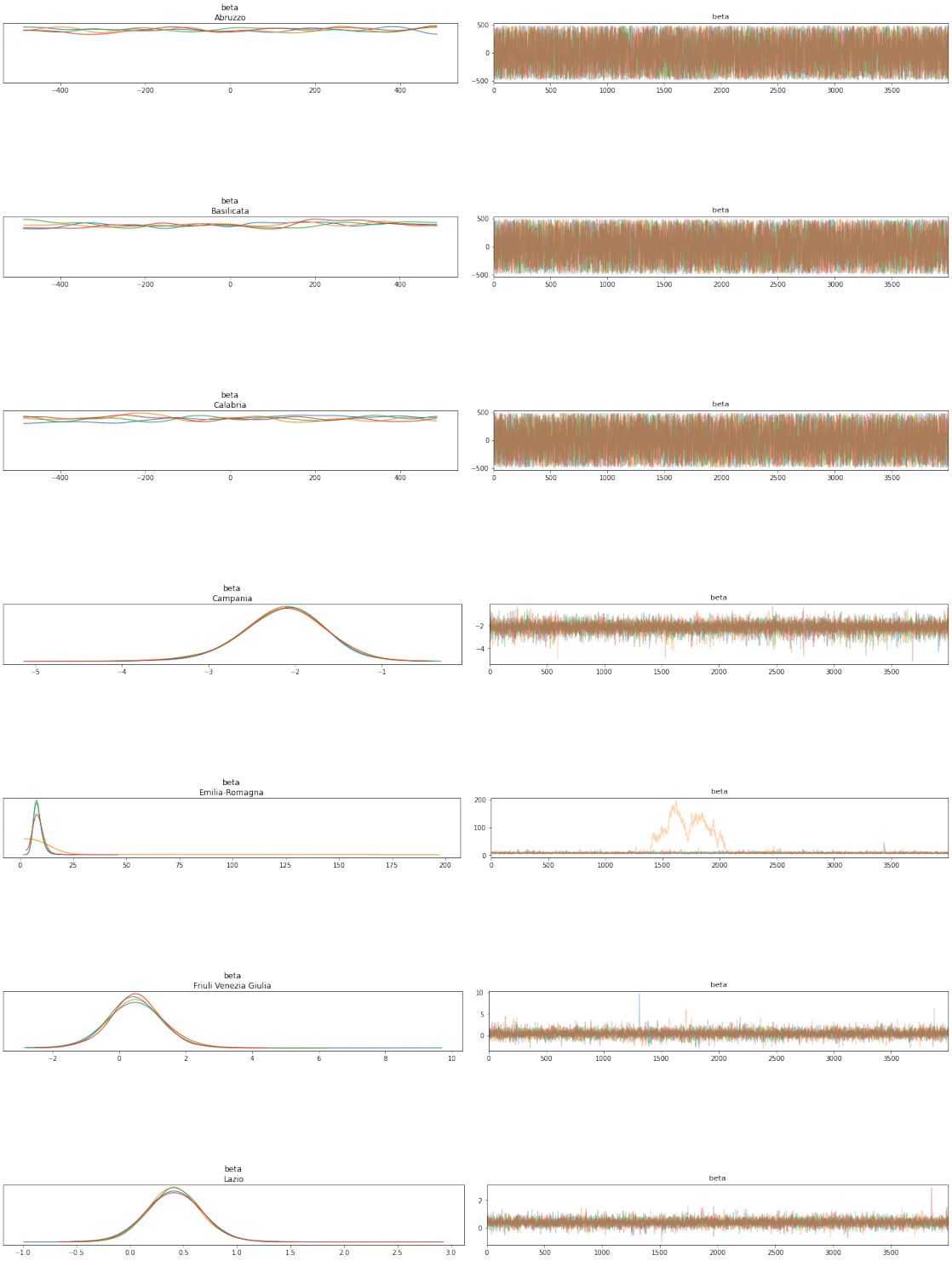
	mean	sd	hdi_0.5%	hdi_99.5%
Abruzzo	0.013	0.003	0.008	0.021
Basilicata	0.013	0.003	0.008	0.021
Calabria	0.013	0.003	0.007	0.021
Campania	0.511	0.111	0.302	0.867
Emilia-Romagna	26.535	7.038	14.940	52.720
Friuli Venezia Giulia	0.542	0.119	0.320	0.930
Lazio	0.497	0.109	0.297	0.840
Liguria	0.288	0.064	0.164	0.496
Lombardia	2.008	0.436	1.161	3.422
Marche	0.013	0.003	0.008	0.021
Molise	0.013	0.003	0.007	0.021
P.A. Bolzano	0.013	0.003	0.008	0.021
P.A. Trento	0.013	0.003	0.008	0.021
Piemonte	0.721	0.155	0.426	1.222
Puglia	0.289	0.064	0.163	0.498
Sardegna	0.013	0.003	0.008	0.021
Sicilia	0.547	0.122	0.311	0.954
Toscana	0.486	0.108	0.282	0.838
Umbria	0.013	0.003	0.008	0.021
Valle d'Aosta	0.013	0.003	0.007	0.021
Veneto	2.393	0.513	1.391	4.030

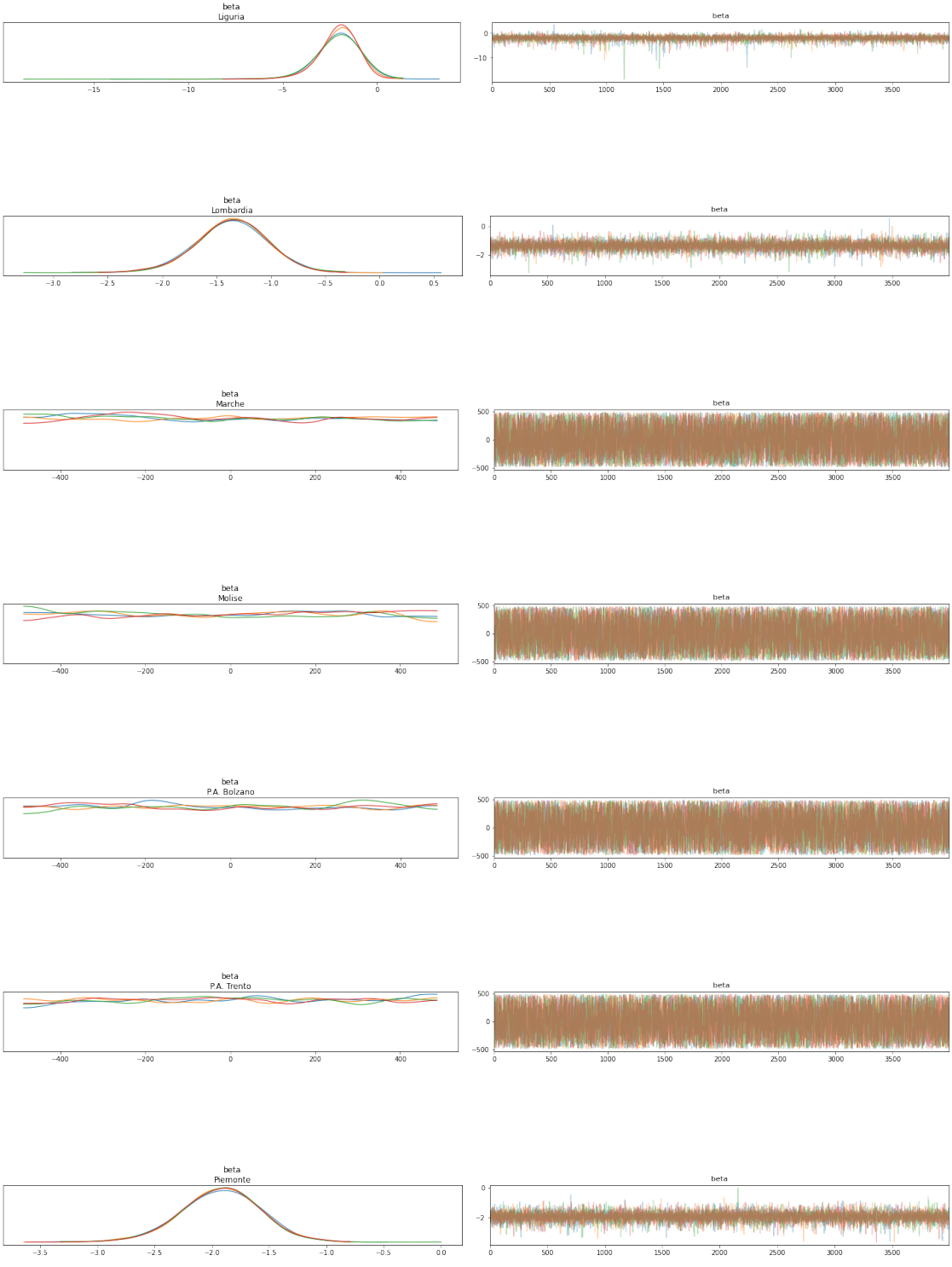
	mcse_mean	mcse_sd	ess_mean	ess_sd	ess_bulk	ess_tail	r_hat
Abruzzo	0.000	0.000	15335.0	15335.0	15177.0	15821.0	1.00
Basilicata	0.000	0.000	15801.0	15705.0	16016.0	15591.0	1.00
Calabria	0.000	0.000	15977.0	15563.0	16031.0	14257.0	1.00
Campania	0.001	0.001	11941.0	11941.0	12109.0	13495.0	1.00
Emilia-Romagna	0.993	0.768	50.0	43.0	106.0	41.0	1.03
Friuli Venezia Giulia	0.001	0.001	11556.0	11292.0	11908.0	13584.0	1.00
Lazio	0.001	0.001	10260.0	9095.0	11333.0	12427.0	1.00
Liguria	0.001	0.000	10796.0	10462.0	11454.0	13220.0	1.00
Lombardia	0.004	0.003	12057.0	11839.0	12453.0	13844.0	1.00
Marche	0.000	0.000	16303.0	16303.0	16216.0	15832.0	1.00
Molise	0.000	0.000	15759.0	15759.0	15691.0	15754.0	1.00
P.A. Bolzano	0.000	0.000	15477.0	15460.0	15291.0	16093.0	1.00
P.A. Trento	0.000	0.000	15709.0	15643.0	15855.0	15916.0	1.00
Piemonte	0.001	0.001	12336.0	12301.0	12624.0	14038.0	1.00
Puglia	0.001	0.000	9775.0	9381.0	10671.0	10620.0	1.00
Sardegna	0.000	0.000	15899.0	15899.0	15829.0	15201.0	1.00
Sicilia	0.001	0.001	7403.0	6220.0	9827.0	7173.0	1.00
Toscana	0.001	0.001	7914.0	7187.0	9717.0	9170.0	1.00
Umbria	0.000	0.000	15967.0	15930.0	15999.0	15406.0	1.00
Valle d'Aosta	0.000	0.000	15879.0	15879.0	15887.0	15846.0	1.00

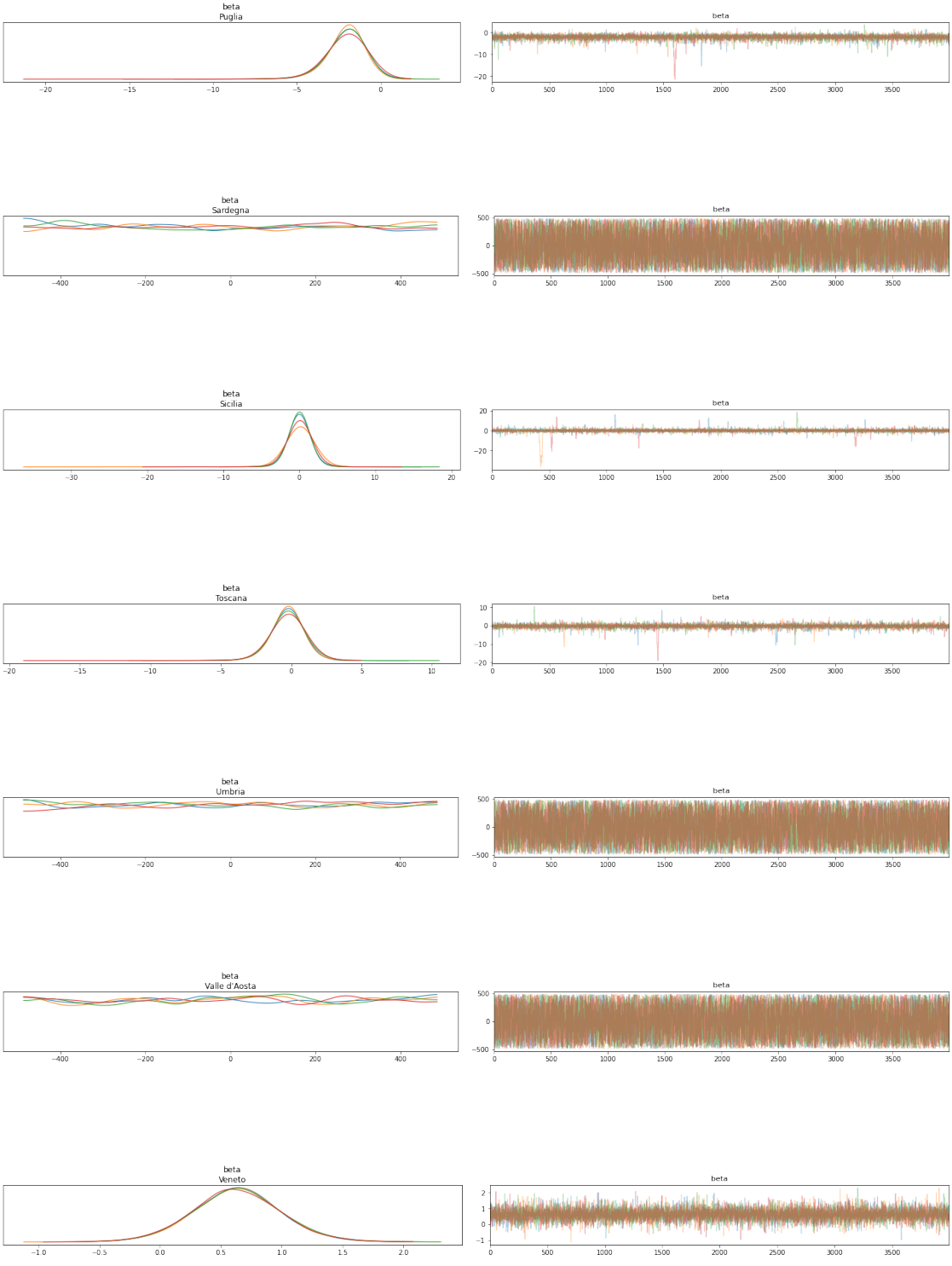
Continued on next page

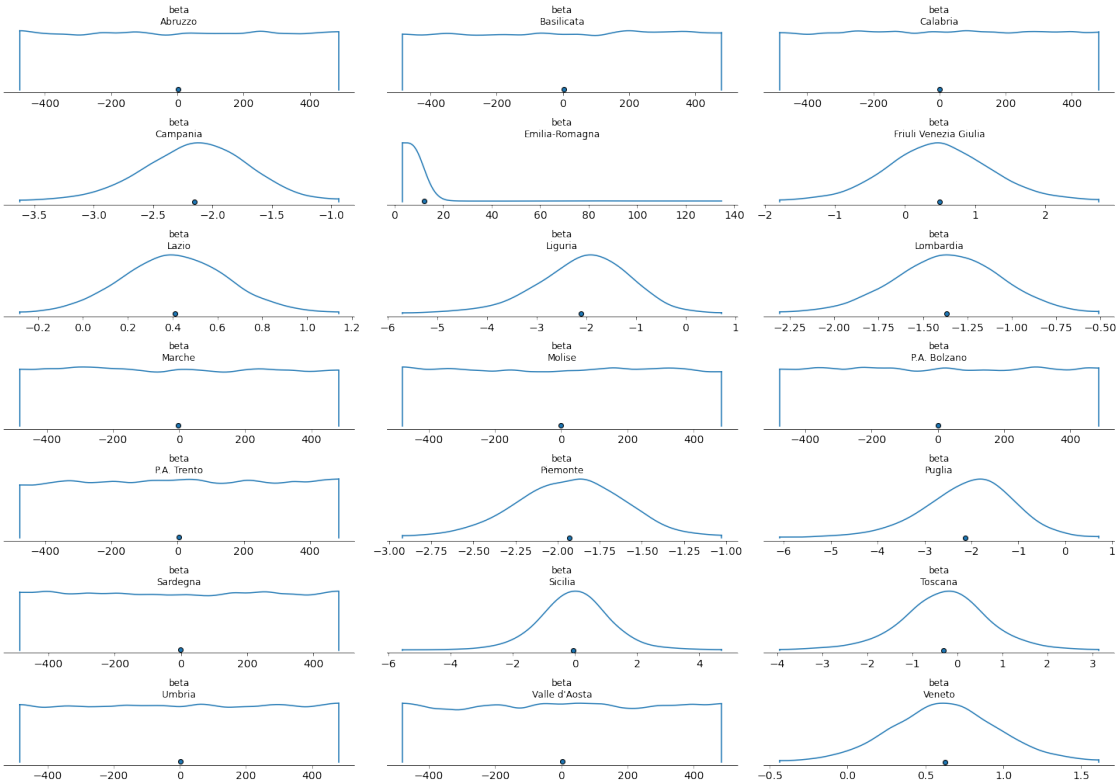
	mcse_mean	mcse_sd	ess_mean	ess_sd	ess_bulk	ess_tail	r_hat
Veneto	0.004	0.003	13659.0	13464.0	14122.0	13777.0	1.00

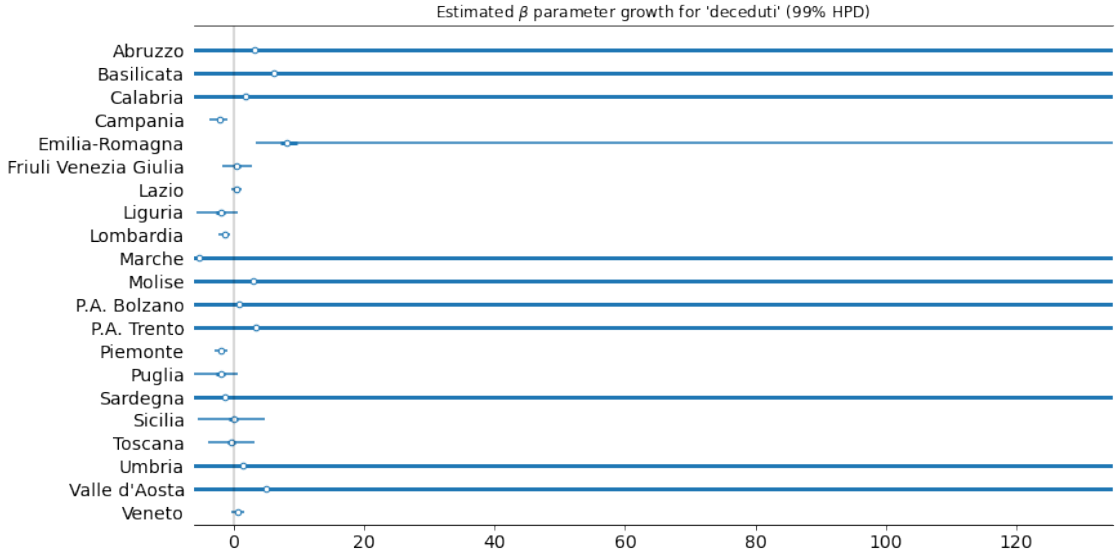
5.1.2 beta




beta
Liguria
beta
beta
Lombardia
beta
beta
Marche
beta
beta
Molise
beta
beta
P.A. Bolzano
beta
beta
P.A. Trento
beta
beta
Piemonte
beta


beta
Puglia
beta
beta
Sardegna
beta
beta
Sicilia
beta
beta
Toscana
beta
beta
Umbria
beta
beta
Valle d'Aosta
beta
beta
Veneto
beta


beta
Abruzzo
beta
Basilicata
beta
Calabria
beta
Campania
beta
Emilia-Romagna
beta
Friuli Venezia Giulia
beta
Lazio
beta
Liguria
beta
Lombardia
beta
Marche
beta
Molise
beta
P.A. Bolzano
beta
P.A. Trento
beta
Piemonte
beta
Puglia
beta
Sardegna
beta
Sicilia
beta
Toscana
beta
Umbria
beta
Valle d'Aosta
beta
Veneto


Estimated β parameter growth for 'deceduti' (99% HPD)
Abruzzo
Basilicata
Calabria
Campania
Emilia-Romagna
Friuli Venezia Giulia
Lazio
Liguria
Lombardia
Marche
Molise
P.A. Bolzano
P.A. Trento
Piemonte
Puglia
Sardegna
Sicilia
Toscana
Umbria
Valle d'Aosta
Veneto
0
20
40
60
80
100
120

	mean	sd	hdi_0.5%	hdi_99.5%
Abruzzo	1.398	281.407	-476.384	486.179
Basilicata	4.207	281.749	-484.156	480.120
Calabria	1.356	280.484	-483.102	481.027
Campania	-2.151	0.478	-3.625	-0.937
Emilia-Romagna	12.306	20.109	3.324	134.823
Friuli Venezia Giulia	0.489	0.815	-1.786	2.759
Lazio	0.413	0.255	-0.282	1.140
Liguria	-2.107	1.112	-5.697	0.729
Lombardia	-1.366	0.325	-2.308	-0.510
Marche	-3.849	281.447	-481.695	481.639
Molise	-0.584	281.418	-477.738	484.461
P.A. Bolzano	0.292	281.263	-476.861	485.321
P.A. Trento	3.380	279.952	-477.449	485.862
Piemonte	-1.932	0.344	-2.919	-1.027
Puglia	-2.130	1.190	-6.087	0.711
Sardegna	-1.441	282.331	-486.954	475.956
Sicilia	-0.059	1.805	-5.535	4.720
Toscana	-0.313	1.168	-3.946	3.137
Umbria	1.558	281.540	-484.190	479.885
Valle d'Aosta	2.924	281.099	-479.052	484.744
Veneto	0.626	0.362	-0.436	1.612

	mcse_mean	mcse_sd	ess_mean	ess_sd	ess_bulk	ess_tail	r_hat
Abruzzo	2.603	1.840	11691.0	11691.0	11905.0	14740.0	1.00
Basilicata	2.617	1.850	11595.0	11595.0	11579.0	13441.0	1.00
Calabria	2.641	1.868	11279.0	11279.0	11362.0	13850.0	1.00
Campania	0.005	0.004	8539.0	7751.0	9090.0	6989.0	1.00
Emilia-Romagna	4.788	3.443	18.0	18.0	88.0	20.0	1.03
Friuli Venezia Giulia	0.009	0.009	8068.0	4502.0	8929.0	7704.0	1.00
Lazio	0.003	0.002	8165.0	5484.0	8878.0	7855.0	1.00
Liguria	0.015	0.014	5566.0	3381.0	8032.0	6057.0	1.00
Lombardia	0.003	0.003	9015.0	8393.0	9291.0	8351.0	1.00
Marche	2.622	1.854	11522.0	11522.0	11687.0	14242.0	1.00
Molise	2.645	1.871	11318.0	11318.0	11525.0	14427.0	1.00
P.A. Bolzano	2.666	1.885	11128.0	11128.0	11319.0	14519.0	1.00
P.A. Trento	2.601	1.839	11583.0	11583.0	11701.0	14768.0	1.00
Piemonte	0.004	0.003	9155.0	8822.0	9427.0	8379.0	1.00
Puglia	0.021	0.022	3241.0	1490.0	7741.0	5092.0	1.00
Sardegna	2.759	1.951	10471.0	10471.0	10762.0	14266.0	1.00
Sicilia	0.052	0.047	1220.0	744.0	6477.0	4059.0	1.00
Toscana	0.019	0.019	3700.0	1910.0	6332.0	4631.0	1.00
Umbria	2.651	1.875	11275.0	11275.0	11401.0	13597.0	1.00
Valle d'Aosta	2.612	1.847	11578.0	11578.0	11795.0	14110.0	1.00

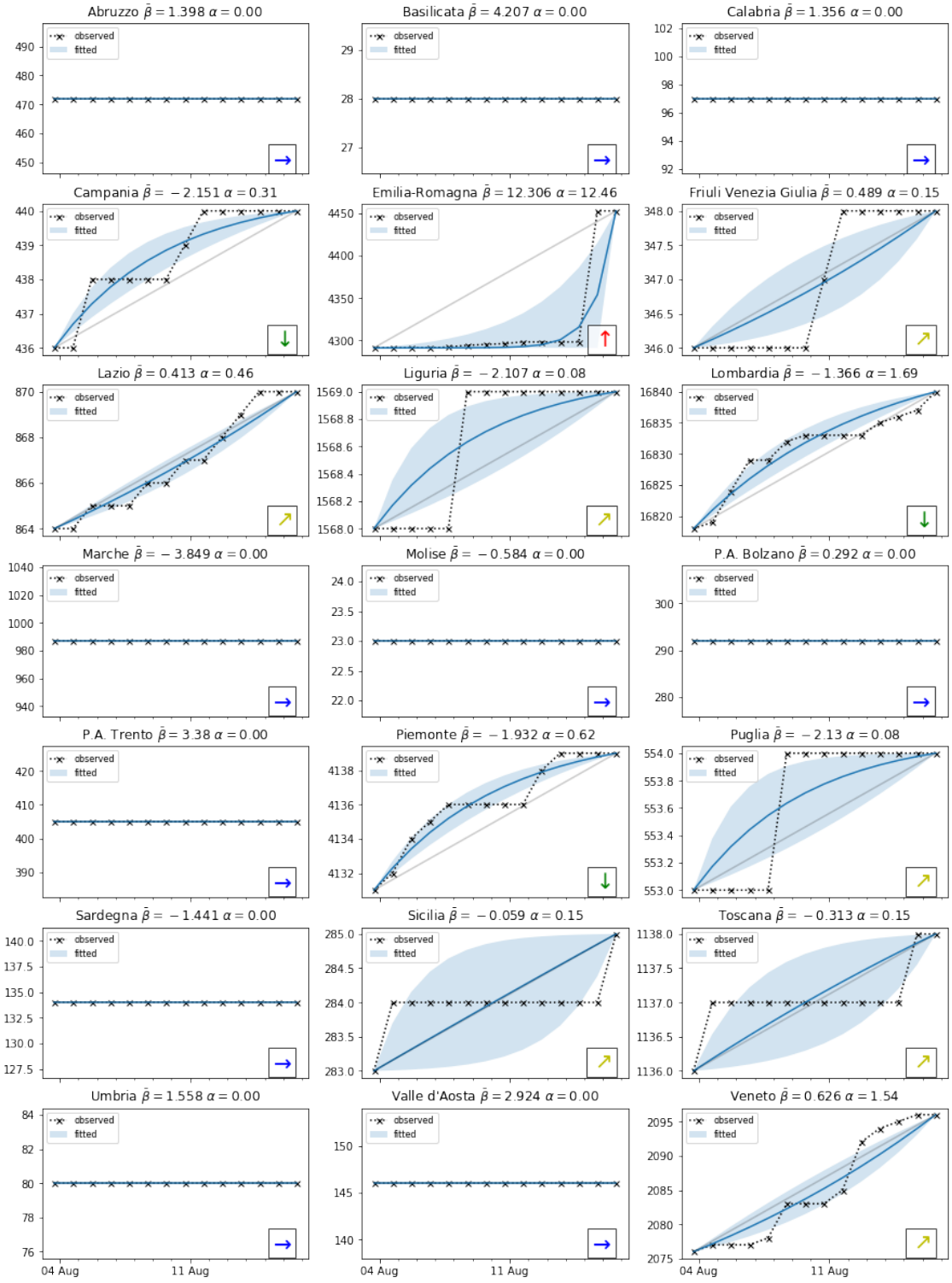
Continued on next page

	mcse_mean	mcse_sd	ess_mean	ess_sd	ess_bulk	ess_tail	r_hat
Veneto	0.004	0.003	9552.0	7738.0	9806.0	8190.0	1.00

5.1.3 fitted

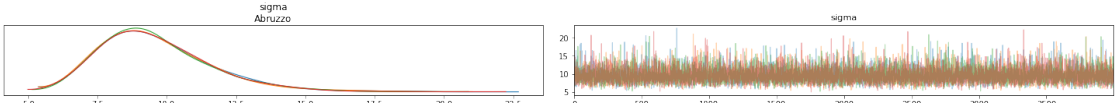
Tabella 7: Bartlett test

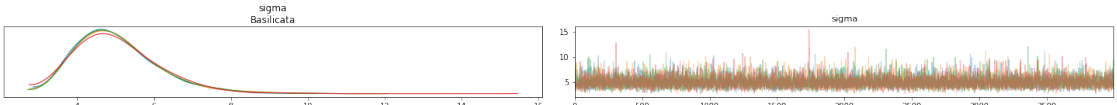
region	p-value
Abruzzo	0.988222
Basilicata	0.646422
Calabria	0.933141
Campania	0.000000
Emilia-Romagna	0.000000
Friuli Venezia Giulia	0.000000
Lazio	0.000000
Liguria	0.000000
Lombardia	0.000000
Marche	0.925069
Molise	0.903841
P.A. Bolzano	0.837604
P.A. Trento	0.703549
Piemonte	0.000000
Puglia	0.000000
Sardegna	0.749946
Sicilia	0.000000
Toscana	0.000000
Umbria	0.898489
Valle d'Aosta	0.978791
Veneto	0.000000

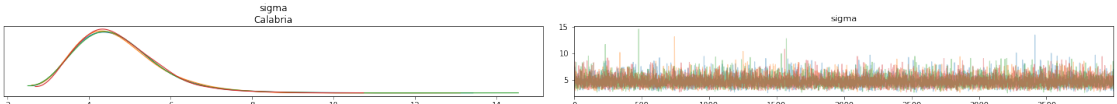

Abruzzo $\bar{\beta} = 1.398$ $\alpha = 0.00$
Basilicata $\bar{\beta} = 4.207$ $\alpha = 0.00$
Calabria $\bar{\beta} = 1.356$ $\alpha = 0.00$
Campania $\bar{\beta} = -2.151$ $\alpha = 0.31$
Emilia-Romagna $\bar{\beta} = 12.306$ $\alpha = 12.46$
Friuli Venezia Giulia $\bar{\beta} = 0.489$ $\alpha = 0.15$
Lazio $\bar{\beta} = 0.413$ $\alpha = 0.46$
Liguria $\bar{\beta} = -2.107$ $\alpha = 0.08$
Lombardia $\bar{\beta} = -1.366$ $\alpha = 1.69$
Marche $\bar{\beta} = -3.849$ $\alpha = 0.00$
Molise $\bar{\beta} = -0.584$ $\alpha = 0.00$
P.A. Bolzano $\bar{\beta} = 0.292$ $\alpha = 0.00$
P.A. Trento $\bar{\beta} = 3.38$ $\alpha = 0.00$
Piemonte $\bar{\beta} = -1.932$ $\alpha = 0.62$
Puglia $\bar{\beta} = -2.13$ $\alpha = 0.08$
Sardegna $\bar{\beta} = -1.441$ $\alpha = 0.00$
Sicilia $\bar{\beta} = -0.059$ $\alpha = 0.15$
Toscana $\bar{\beta} = -0.313$ $\alpha = 0.15$
Umbria $\bar{\beta} = 1.558$ $\alpha = 0.00$
Valle d'Aosta $\bar{\beta} = 2.924$ $\alpha = 0.00$
Veneto $\bar{\beta} = 0.626$ $\alpha = 1.54$
observed
fitted
04 Aug
11 Aug

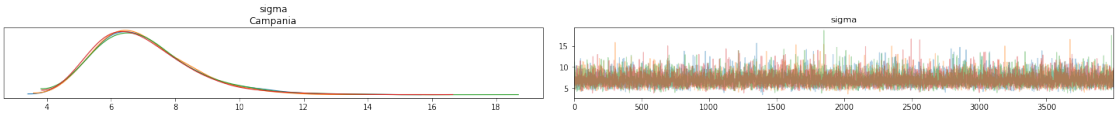
5.2 totale_casi

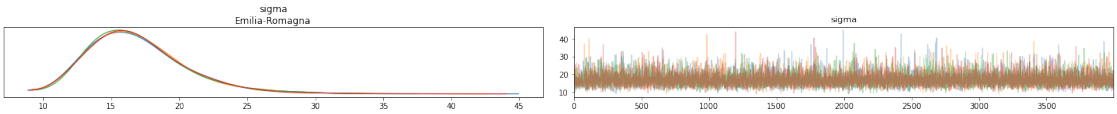
5.2.1 sigma

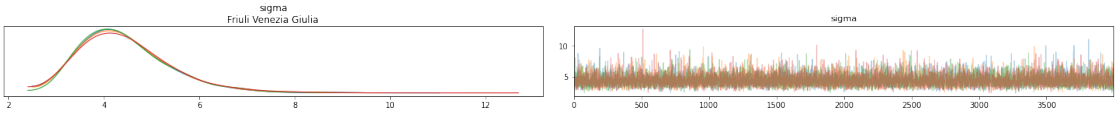


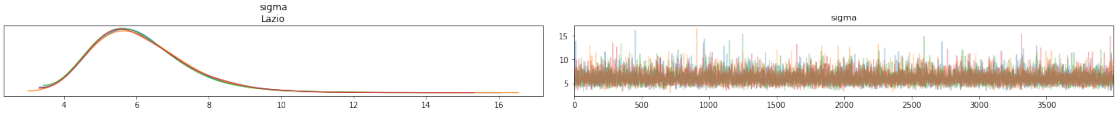


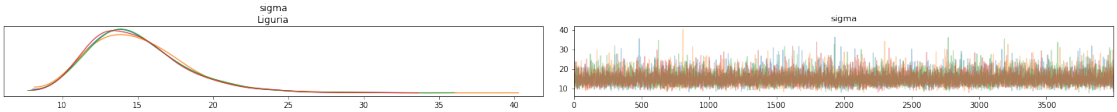


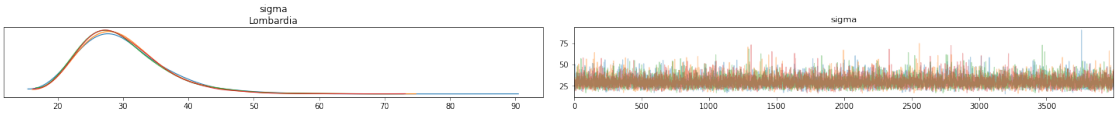


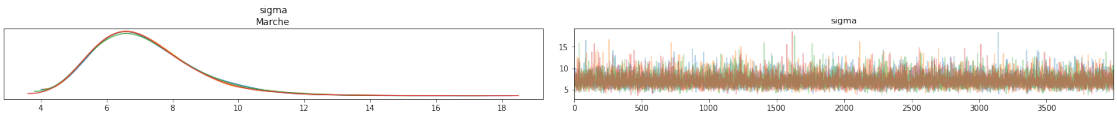


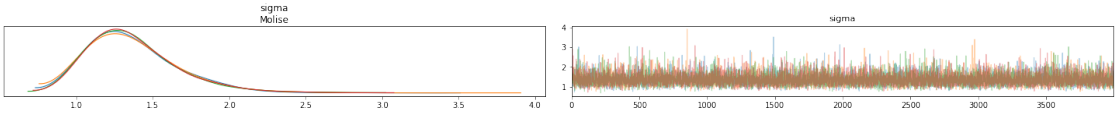


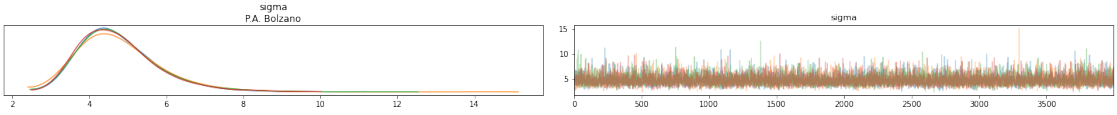


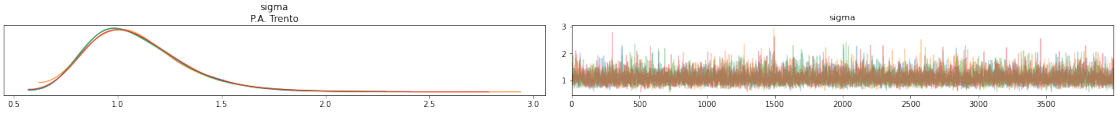

sigma
Liguria
sigma

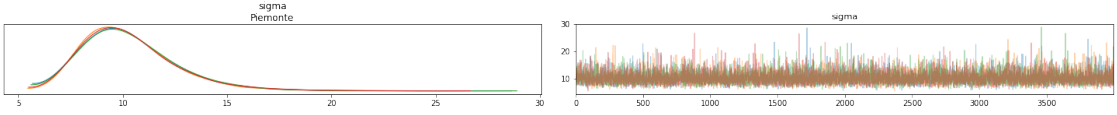

sigma
Lombardia
sigma

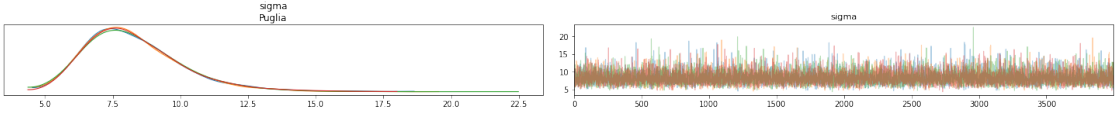

sigma
Marche
sigma

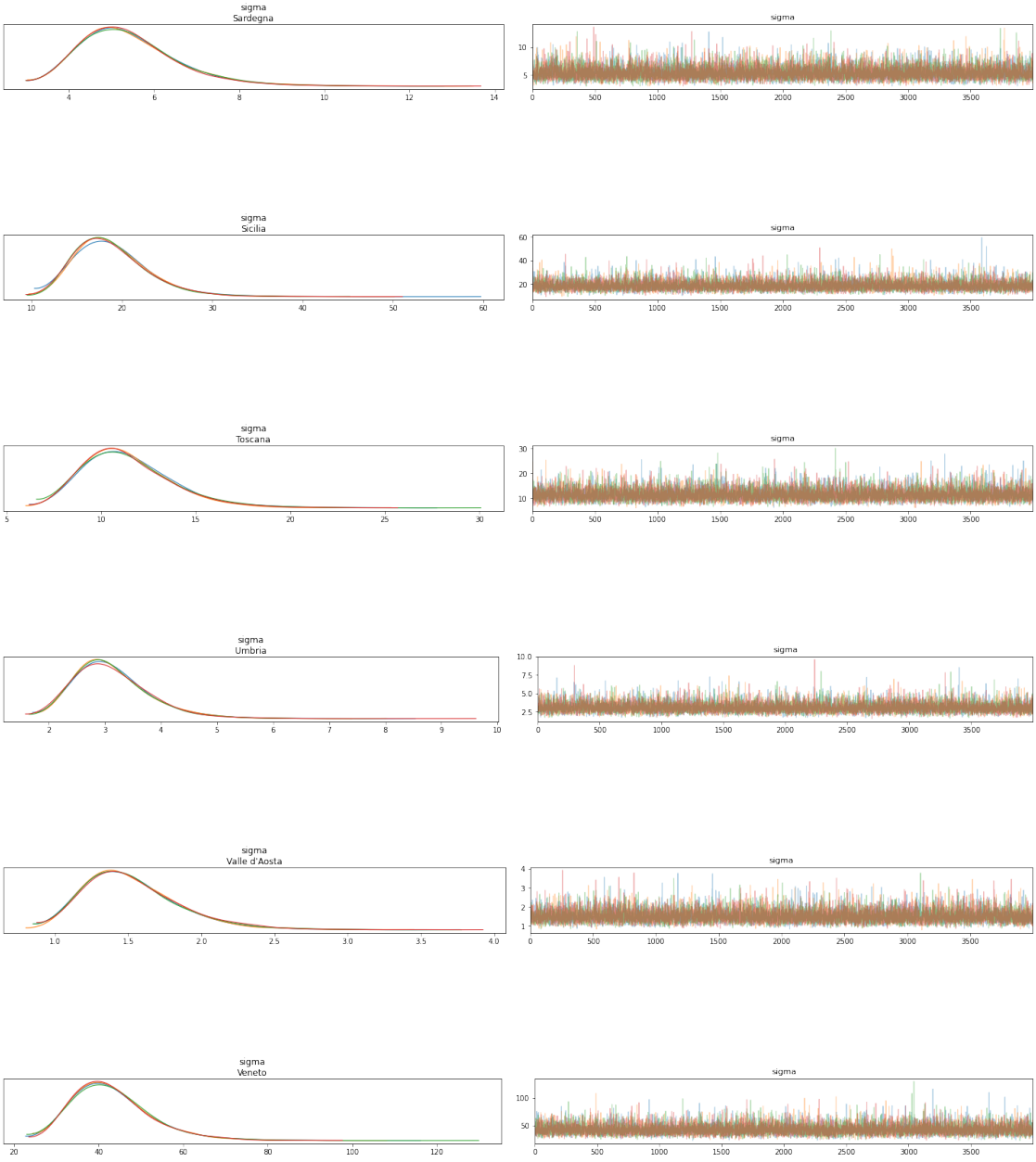

sigma
Molise
sigma

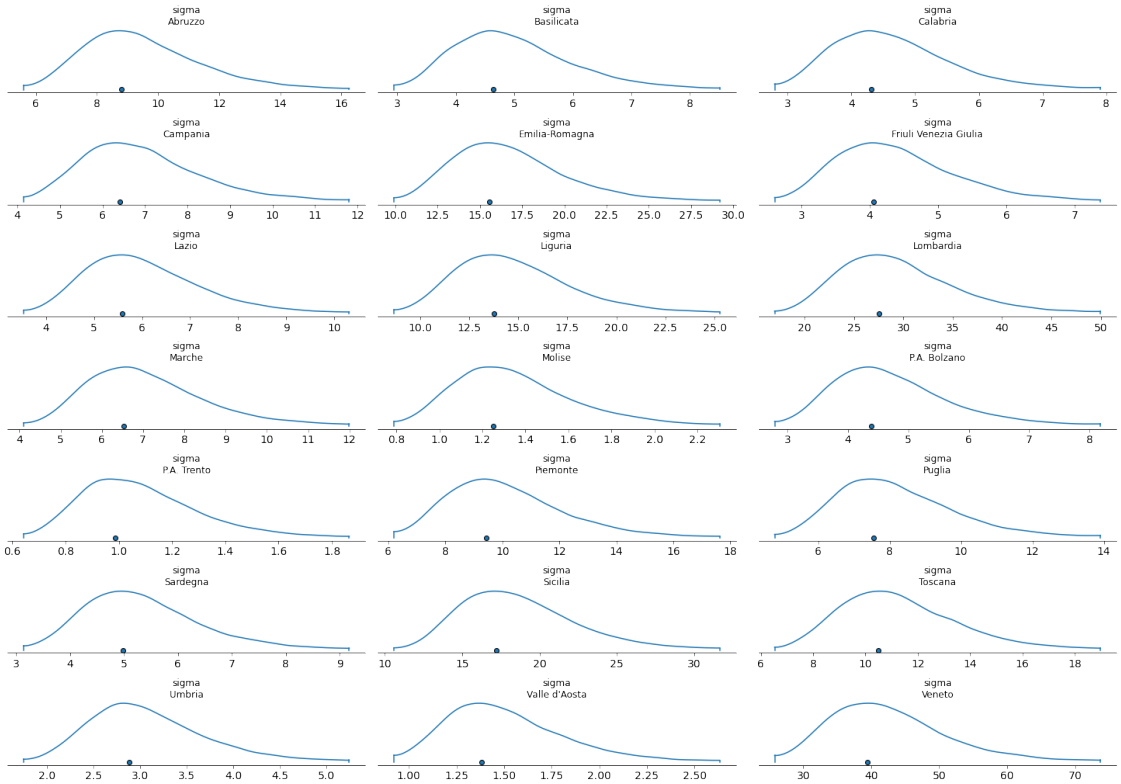

sigma
P.A. Bolzano
sigma

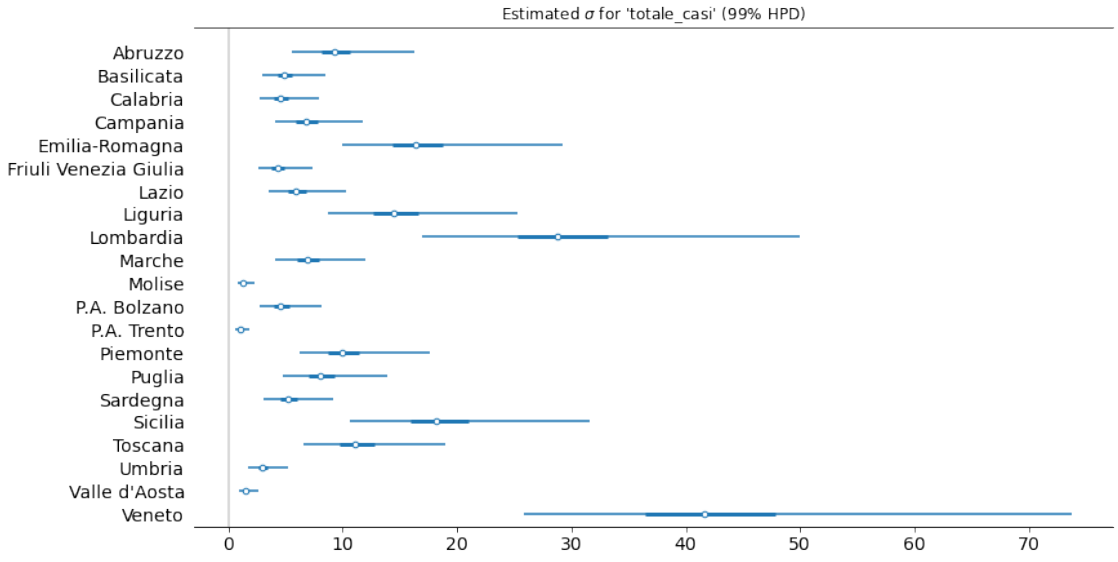

sigma
P.A. Trento
sigma


sigma
Piemonte
sigma


sigma
Puglia
sigma


sigma
Sardegna
sigma
sigma
Sicilia
sigma
sigma
Toscana
sigma
sigma
Umbria
sigma
sigma
Valle d'Aosta
sigma
sigma
Veneto
sigma


sigma
Abruzzo
sigma
Basilicata
sigma
Calabria
sigma
Campania
sigma
Emilia-Romagna
sigma
Friuli Venezia Giulia
sigma
Lazio
sigma
Liguria
sigma
Lombardia
sigma
Marche
sigma
Molise
sigma
P.A. Bolzano
sigma
P.A. Trento
sigma
Piemonte
sigma
Puglia
sigma
Sardegna
sigma
Sicilia
sigma
Toscana
sigma
Umbria
sigma
Valle d'Aosta
sigma
Veneto


Estimated σ for 'totale_casi' (99% HPD)
Abruzzo
Basilicata
Calabria
Campania
Emilia-Romagna
Friuli Venezia Giulia
Lazio
Liguria
Lombardia
Marche
Molise
P.A. Bolzano
P.A. Trento
Piemonte
Puglia
Sardegna
Sicilia
Toscana
Umbria
Valle d'Aosta
Veneto
0
10
20
30
40
50
60
70

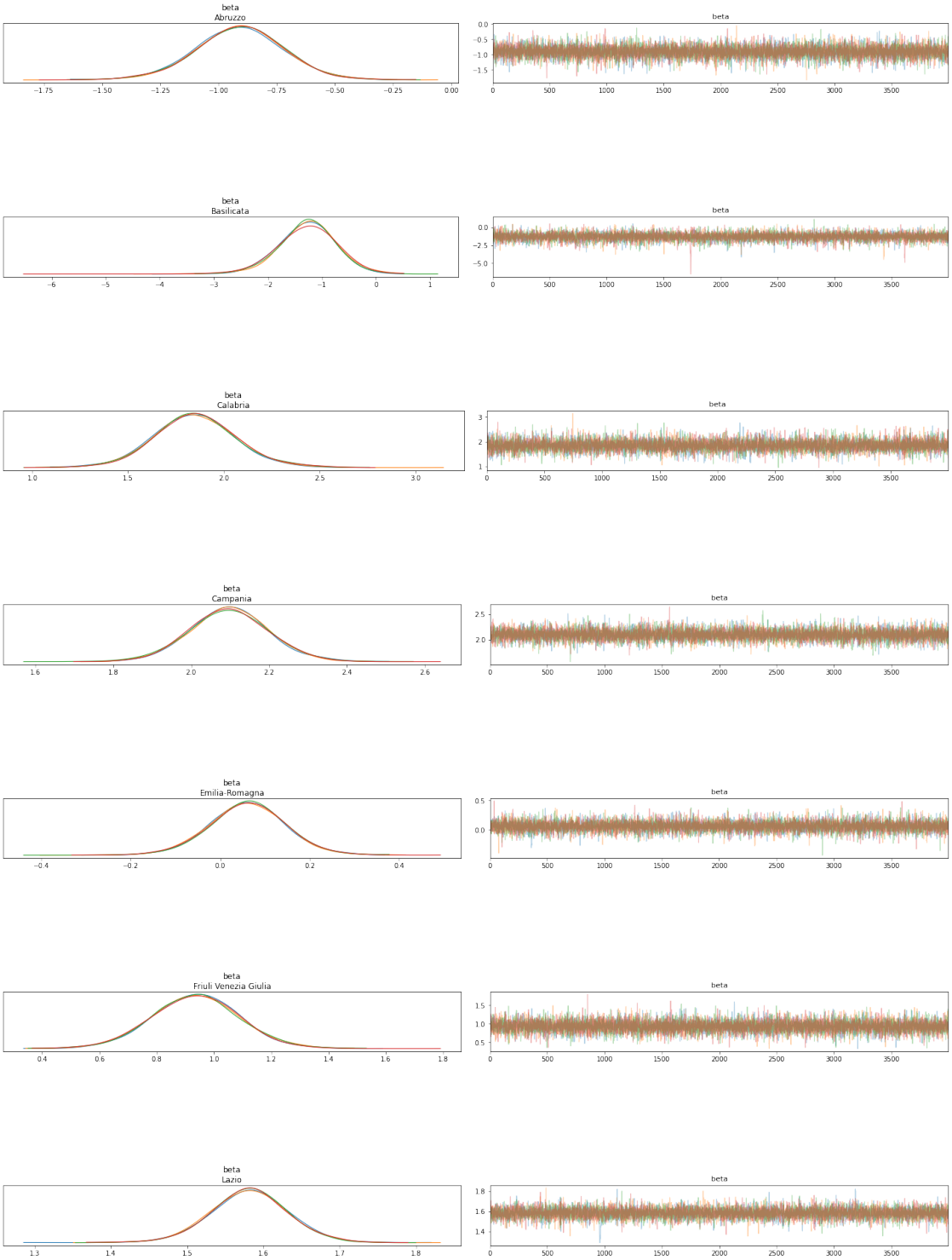
	mean	sd	hdi_0.5%	hdi_99.5%
Abruzzo	9.576	2.039	5.599	16.253
Basilicata	5.034	1.085	2.944	8.507
Calabria	4.682	1.000	2.794	7.909
Campania	6.997	1.502	4.141	11.804
Emilia-Romagna	16.928	3.647	9.911	29.199
Friuli Venezia Giulia	4.432	0.943	2.606	7.376
Lazio	6.103	1.311	3.531	10.313
Liguria	14.964	3.227	8.671	25.257
Lombardia	29.764	6.420	16.962	49.970
Marche	7.144	1.525	4.102	11.994
Molise	1.361	0.297	0.789	2.303
P.A. Bolzano	4.762	1.033	2.788	8.178
P.A. Trento	1.092	0.236	0.642	1.865
Piemonte	10.272	2.238	6.194	17.625
Puglia	8.272	1.789	4.776	13.904
Sardegna	5.407	1.166	3.134	9.177
Sicilia	18.853	4.086	10.582	31.650
Toscana	11.411	2.421	6.530	18.980
Umbria	3.115	0.677	1.747	5.245
Valle d'Aosta	1.525	0.333	0.922	2.632
Veneto	43.018	9.340	25.797	73.731

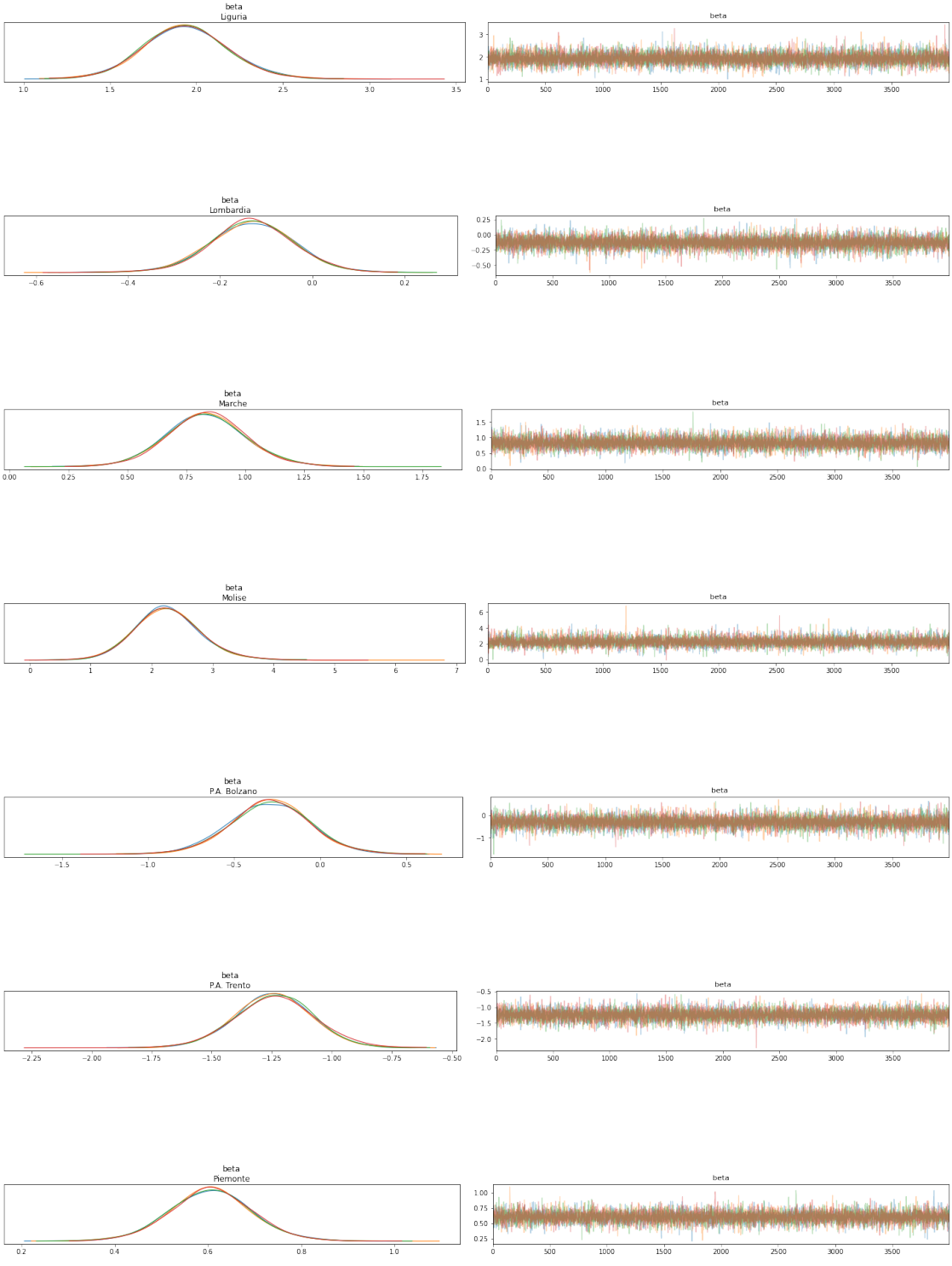
	mcse_mean	mcse_sd	ess_mean	ess_sd	ess_bulk	ess_tail	r_hat
Abruzzo	0.018	0.013	12743.0	12667.0	13465.0	14352.0	1.0
Basilicata	0.010	0.007	11700.0	11174.0	12471.0	13289.0	1.0
Calabria	0.009	0.006	12851.0	12851.0	12947.0	15019.0	1.0
Campania	0.013	0.009	13048.0	13048.0	13118.0	14091.0	1.0
Emilia-Romagna	0.034	0.024	11811.0	11749.0	12174.0	13252.0	1.0
Friuli Venezia Giulia	0.008	0.006	13605.0	13553.0	13634.0	14657.0	1.0
Lazio	0.013	0.009	10909.0	10384.0	11846.0	12271.0	1.0
Liguria	0.029	0.021	12534.0	12274.0	13279.0	13632.0	1.0
Lombardia	0.059	0.042	11922.0	11662.0	12348.0	13630.0	1.0
Marche	0.013	0.010	12858.0	12741.0	13254.0	14279.0	1.0
Molise	0.003	0.002	11788.0	11417.0	12831.0	13320.0	1.0
P.A. Bolzano	0.009	0.007	12599.0	12350.0	13059.0	13917.0	1.0
P.A. Trento	0.002	0.001	12682.0	12540.0	12849.0	14016.0	1.0
Piemonte	0.020	0.014	12297.0	12297.0	12571.0	13972.0	1.0
Puglia	0.017	0.012	11283.0	11283.0	11465.0	13373.0	1.0
Sardegna	0.010	0.007	13241.0	13241.0	13292.0	14755.0	1.0
Sicilia	0.037	0.027	12009.0	11725.0	12498.0	13380.0	1.0
Toscana	0.022	0.016	11861.0	11839.0	12035.0	13543.0	1.0
Umbria	0.006	0.004	12054.0	12054.0	12139.0	13823.0	1.0
Valle d'Aosta	0.003	0.002	9872.0	9781.0	10363.0	11762.0	1.0

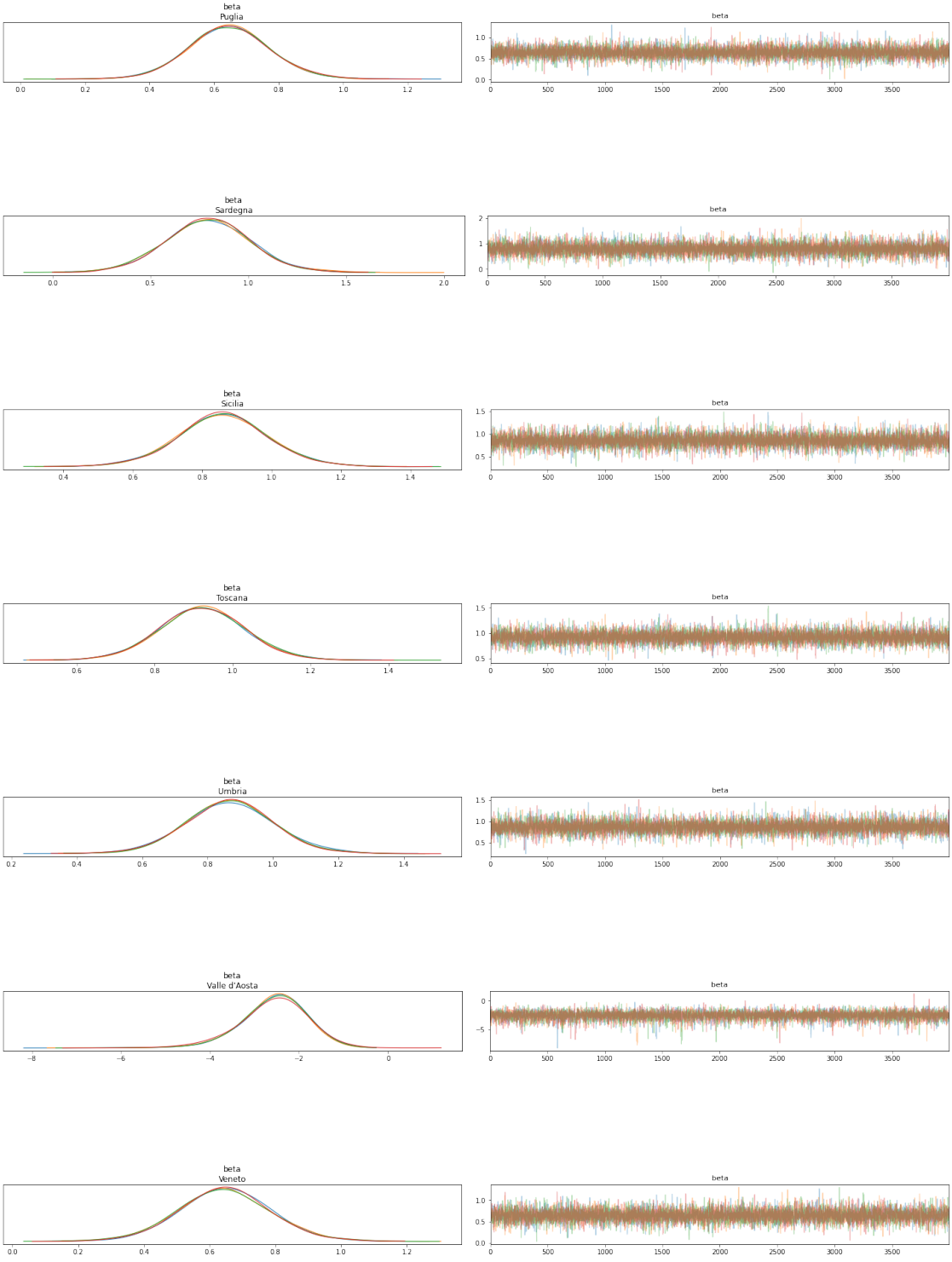
Continued on next page

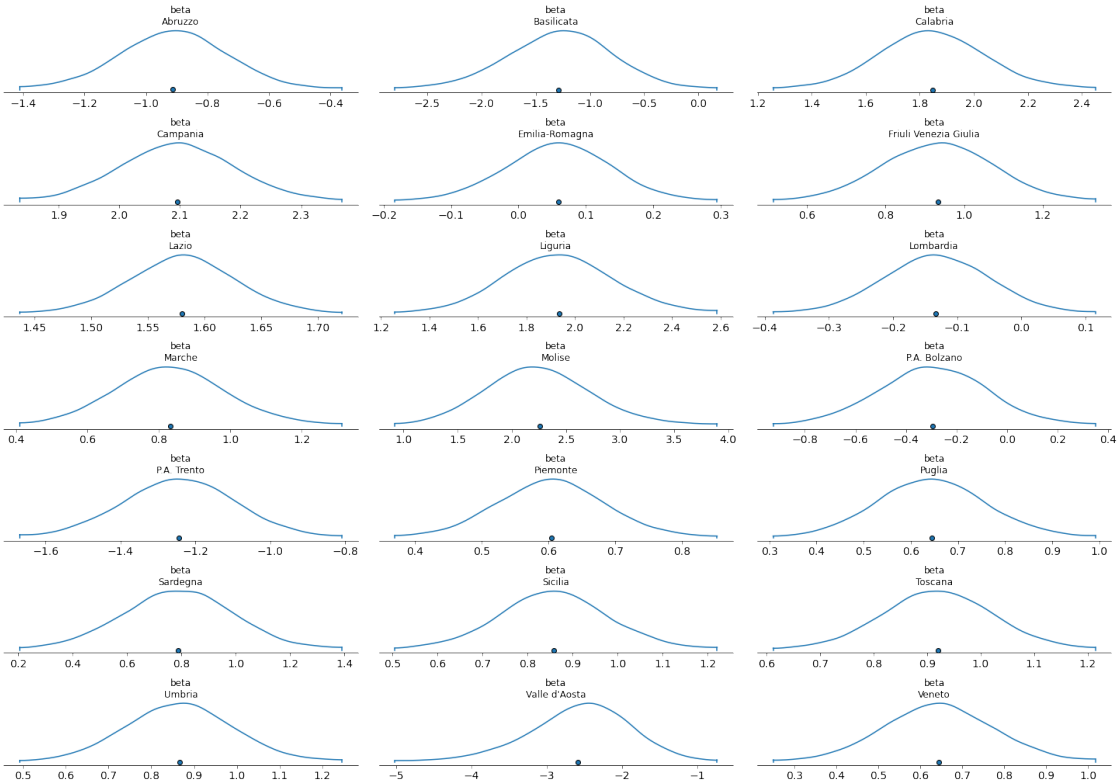
	mcse_mean	mcse_sd	ess_mean	ess_sd	ess_bulk	ess_tail	r_hat
Veneto	0.083	0.059	12786.0	12701.0	13077.0	13954.0	1.0

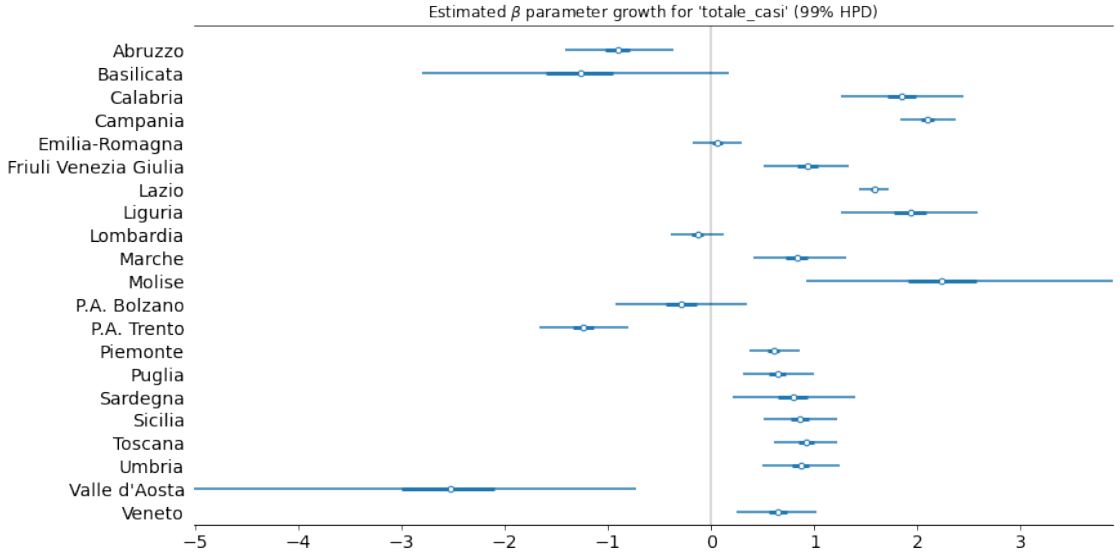
5.2.2 beta




beta
Liguria
beta
beta
Lombardia
beta
beta
Marche
beta
beta
Molise
beta
beta
P.A. Bolzano
beta
beta
P.A. Trento
beta
beta
Piemonte
beta


beta
Puglia
beta
beta
Sardegna
beta
beta
Sicilia
beta
beta
Toscana
beta
beta
Umbria
beta
beta
Valle d'Aosta
beta
beta
Veneto
beta


beta
Abruzzo
beta
Basilicata
beta
Calabria
beta
Campania
beta
Emilia-Romagna
beta
Friuli Venezia Giulia
beta
Lazio
beta
Liguria
beta
Lombardia
beta
Marche
beta
Molise
beta
P.A. Bolzano
beta
P.A. Trento
beta
Piemonte
beta
Puglia
beta
Sardegna
beta
Sicilia
beta
Toscana
beta
Umbria
beta
Valle d'Aosta
beta
Veneto


Estimated β parameter growth for 'totale_casi' (99% HPD)
Abruzzo
Basilicata
Calabria
Campania
Emilia-Romagna
Friuli Venezia Giulia
Lazio
Liguria
Lombardia
Marche
Molise
P.A. Bolzano
P.A. Trento
Piemonte
Puglia
Sardegna
Sicilia
Toscana
Umbria
Valle d'Aosta
Veneto

	mean	sd	hdi_0.5%	hdi_99.5%
Abruzzo	-0.912	0.188	-1.411	-0.362
Basilicata	-1.289	0.530	-2.798	0.173
Calabria	1.848	0.213	1.257	2.450
Campania	2.096	0.100	1.836	2.367
Emilia-Romagna	0.059	0.085	-0.183	0.295
Friuli Venezia Giulia	0.933	0.151	0.514	1.334
Lazio	1.580	0.050	1.437	1.721
Liguria	1.935	0.243	1.257	2.586
Lombardia	-0.133	0.090	-0.387	0.115
Marche	0.832	0.166	0.411	1.312
Molise	2.256	0.531	0.920	3.896
P.A. Bolzano	-0.295	0.231	-0.925	0.349
P.A. Trento	-1.244	0.159	-1.669	-0.809
Piemonte	0.605	0.087	0.370	0.853
Puglia	0.644	0.124	0.308	0.992
Sardegna	0.790	0.218	0.206	1.391
Sicilia	0.858	0.130	0.506	1.221
Toscana	0.921	0.110	0.613	1.214
Umbria	0.866	0.134	0.492	1.246
Valle d'Aosta	-2.589	0.756	-5.015	-0.734
Veneto	0.644	0.140	0.248	1.017

	mcse_mean	mcse_sd	ess_mean	ess_sd	ess_bulk	ess_tail	r_hat
Abruzzo	0.002	0.001	8756.0	8506.0	8980.0	8207.0	1.0
Basilicata	0.006	0.005	7840.0	5485.0	9077.0	7215.0	1.0
Calabria	0.002	0.002	8723.0	8664.0	8886.0	8379.0	1.0
Campania	0.001	0.001	9429.0	9424.0	9576.0	8184.0	1.0
Emilia-Romagna	0.001	0.001	8977.0	7108.0	9239.0	8304.0	1.0
Friuli Venezia Giulia	0.002	0.001	9627.0	9627.0	9721.0	8493.0	1.0
Lazio	0.001	0.000	8301.0	8301.0	8861.0	7283.0	1.0
Liguria	0.003	0.002	9294.0	9023.0	9519.0	8081.0	1.0
Lombardia	0.001	0.001	9085.0	7421.0	9407.0	8010.0	1.0
Marche	0.002	0.001	9387.0	9117.0	9505.0	8384.0	1.0
Molise	0.006	0.004	8925.0	7997.0	9375.0	8178.0	1.0
P.A. Bolzano	0.002	0.002	10003.0	8291.0	10159.0	8483.0	1.0
P.A. Trento	0.002	0.001	9709.0	9612.0	9789.0	9238.0	1.0
Piemonte	0.001	0.001	9052.0	8783.0	9296.0	7857.0	1.0
Puglia	0.001	0.001	9113.0	8969.0	9339.0	7919.0	1.0
Sardegna	0.002	0.002	9884.0	9670.0	9933.0	8920.0	1.0
Sicilia	0.001	0.001	8791.0	8714.0	9007.0	7967.0	1.0
Toscana	0.001	0.001	9554.0	9472.0	9651.0	8701.0	1.0
Umbria	0.001	0.001	9770.0	9658.0	9958.0	8519.0	1.0
Valle d'Aosta	0.009	0.008	6478.0	4869.0	8319.0	6085.0	1.0

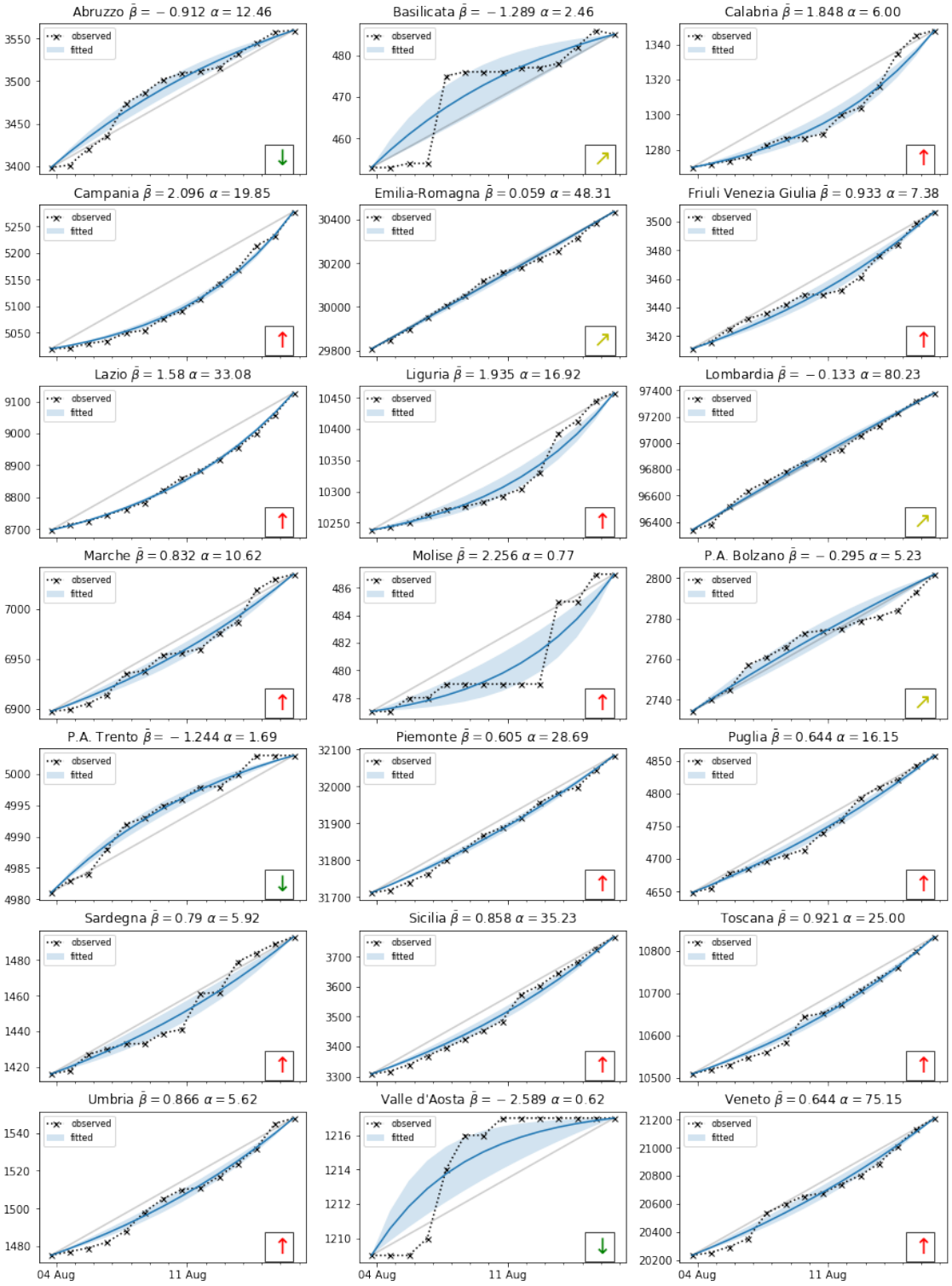
Continued on next page

	mcse_mean	mcse_sd	ess_mean	ess_sd	ess_bulk	ess_tail	r_hat
Veneto	0.001	0.001	9597.0	9447.0	9742.0	8521.0	1.0

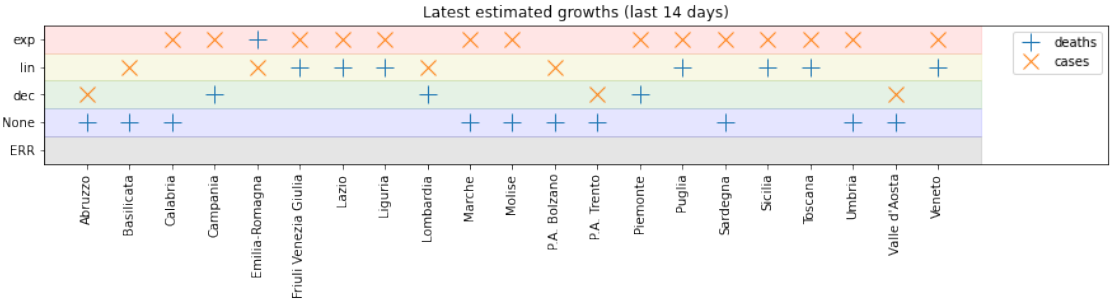
5.2.3 fitted

Tabella 12: Bartlett test

region	p-value
Abruzzo	0.0
Basilicata	0.0
Calabria	0.0
Campania	0.0
Emilia-Romagna	0.0
Friuli Venezia Giulia	0.0
Lazio	0.0
Liguria	0.0
Lombardia	0.0
Marche	0.0
Molise	0.0
P.A. Bolzano	0.0
P.A. Trento	0.0
Piemonte	0.0
Puglia	0.0
Sardegna	0.0
Sicilia	0.0
Toscana	0.0
Umbria	0.0
Valle d'Aosta	0.0
Veneto	0.0


Abruzzo $\bar{\beta} = -0.912$ $\alpha = 12.46$
Basilicata $\bar{\beta} = -1.289$ $\alpha = 2.46$
Calabria $\bar{\beta} = 1.848$ $\alpha = 6.00$
Campania $\bar{\beta} = 2.096$ $\alpha = 19.85$
Emilia-Romagna $\bar{\beta} = 0.059$ $\alpha = 48.31$
Friuli Venezia Giulia $\bar{\beta} = 0.933$ $\alpha = 7.38$
Lazio $\bar{\beta} = 1.58$ $\alpha = 33.08$
Liguria $\bar{\beta} = 1.935$ $\alpha = 16.92$
Lombardia $\bar{\beta} = -0.133$ $\alpha = 80.23$
Marche $\bar{\beta} = 0.832$ $\alpha = 10.62$
Molise $\bar{\beta} = 2.256$ $\alpha = 0.77$
P.A. Bolzano $\bar{\beta} = -0.295$ $\alpha = 5.23$
P.A. Trento $\bar{\beta} = -1.244$ $\alpha = 1.69$
Piemonte $\bar{\beta} = 0.605$ $\alpha = 28.69$
Puglia $\bar{\beta} = 0.644$ $\alpha = 16.15$
Sardegna $\bar{\beta} = 0.79$ $\alpha = 5.92$
Sicilia $\bar{\beta} = 0.858$ $\alpha = 35.23$
Toscana $\bar{\beta} = 0.921$ $\alpha = 25.00$
Umbria $\bar{\beta} = 0.866$ $\alpha = 5.62$
Valle d'Aosta $\bar{\beta} = -2.589$ $\alpha = 0.62$
Veneto $\bar{\beta} = 0.644$ $\alpha = 75.15$
observed
fitted
04 Aug
11 Aug

5.3 Summary



Growths summary:

	deceduti			totale_casi		
	growth	beta	alpha	growth	beta	alpha
Abruzzo	None	0	0	dec	-0.91	12.46
Basilicata	None	0	0	lin	-1.29	2.46
Calabria	None	0	0	exp	1.85	6.00
Campania	dec	-2.15	0.31	exp	2.10	19.85
Emilia-Romagna	exp	12.31	12.46	lin	0.06	48.31
Friuli Venezia Giulia	lin	0.49	0.15	exp	0.93	7.38
Lazio	lin	0.41	0.46	exp	1.58	33.08
Liguria	lin	-2.11	0.08	exp	1.94	16.92
Lombardia	dec	-1.37	1.69	lin	-0.13	80.23
Marche	None	0	0	exp	0.83	10.62
Molise	None	0	0	exp	2.26	0.77
P.A. Bolzano	None	0	0	lin	-0.29	5.23
P.A. Trento	None	0	0	dec	-1.24	1.69
Piemonte	dec	-1.93	0.62	exp	0.60	28.69
Puglia	lin	-2.13	0.08	exp	0.64	16.15
Sardegna	None	0	0	exp	0.79	5.92
Sicilia	lin	-0.06	0.15	exp	0.86	35.23
Toscana	lin	-0.31	0.15	exp	0.92	25.00
Umbria	None	0	0	exp	0.87	5.62
Valle d'Aosta	None	0	0	dec	-2.59	0.62
Veneto	lin	0.63	1.54	exp	0.64	75.15

	ERR	None	dec	lin	exp
deaths	0	10	3	7	1
cases	0	0	3	4	14

6 CONCLUSIONS

The proposed unified exponential curve function can correctly describe the four supposed cumulative data growths (none, linear, exponential increasing decay, exponential growth) with 99% credible interval.

For cumulative total cases

- 14 regions 66.67% are likely exponentially growing
- 4 regions 19.05% are likely linearly growing
- 3 regions 14.29% are likely exponentially decaying
- 0 regions 0.00% are likely constant

For cumulative total deaths:

- 1 regions 4.76% are likely exponentially growing
- 7 regions 33.33% are likely linearly growing
- 3 regions 14.29% are likely exponentially decaying
- 10 regions 47.62% are likely constant

In 67% regions, cases are growing exponetially and in 14% the situation is likely solved or about to resolution (no growth or exponential decay).

On the other hand, in 62% regions deaths are constant or decaying and 5% are showing an exponential growth of deaths.

Linearly growing regions (19% for cases and 33% for deaths) situation is not severe but likely not currently resolving.